A Co-Channel Interference Cancellation Technique Using Orthogonal Convolutional Codes
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Abstract—This paper proposes a new parallel co-channel interference cancellation technique which utilizes orthogonal convolutional codes. Co-channel interference (CCI) limits the performance of a spread spectrum multiple access communication link. Several CCI cancellation techniques have been proposed to remove this interference. Of particular interest are techniques which do not require the receiver to have knowledge of the cross-correlation between user sequences. These techniques reconstruct the CCI based on the initial decisions regarding the signals from the other users. However, these techniques leave residual interference after cancellation caused by errors in these initial decisions. To improve the initial decisions and reduce the residual interference, our proposed scheme utilizes the error correcting capability of orthogonal convolutional codes. This paper evaluates the performance of this scheme. We show that, given a processing gain of 128 for up to about 40 users, the performance of the proposed CCI canceller approaches the performance of a system without multi-user interference. We also show that the proposed CCI canceller offers an improvement in capacity by a factor of 1.5 ~ 3 over that of a conventional canceller.

I. INTRODUCTION

SPREAD SPECTRUM techniques have recently received much attention in wireless communication applications such as the low-cost wireless local area network (LAN). This is in large part due to the fact that spread spectrum techniques have superior multi-access capability, anti-multipath fading capability, and anti-jamming capability [1].

There are three basic multiple access schemes for wireless communications: frequency division multiple access (FDMA), time division multiple access (TDMA), and code division multiple access (CDMA) [2]. CDMA capacity is interference limited, unlike FDMA, and TDMA capacities, which are primarily bandwidth limited [3]. It has been suggested that CDMA could be used to increase the number of channels per unit bandwidth under certain conditions [2], [3]. The promise of an increase in capacity has made CDMA very attractive, for instance, in cellular communications where no additional spectrum will be allocated for digital cellular [2]. CDMA has also been considered for satellite and other wireless applications.

To fully realize the capacity potential of CDMA, multi-user detection has been considered [4]. In [5], the optimum multi-user detector for asynchronous CDMA was derived and analyzed. It consists of a matched filter front end followed by a Viterbi algorithm. Although the optimum detector significantly outperforms the conventional single user detector, its computational complexity grows exponentially with the number of users.

Less complex suboptimum multi-user detectors have also been considered. In [6], sequential decoding is applied instead of the Viterbi algorithm. In [7]–[10], the decorrelating detector, which multiplies an inverse cross-correlation matrix with the matched filter outputs, has been investigated. In [11], the decorrelator is combined with a decision-feedback detector. Although the performance of these multi-user detectors is close to that of the optimum detector and they involve a reasonable amount of computation, these detectors must calculate the inverse cross-correlation matrix. In [12], the minimum mean-square error (MMSE) detector which calculates the inverse matrix adaptively, was proposed. This detector outperforms the decorrelating detector when background noise is the dominant factor limiting the performance. However, it requires a training sequence. In [13] and [14], tentative-decision based multi-user detectors have been investigated. These detectors have a multistage structure, where the first stage consists of a bank of conventional detectors. The second and third stages assume that the previous decisions are correct, calculate the co-channel interference (CCI) caused by undesired users’ signals, and remove the residual interference from the correlator output of the desired user’s signal.

Unfortunately, these suboptimum techniques are not practical if long pseudo noise (PN) sequences are used as signature sequences to separate the users [15]. This is because these techniques assume that the receiver has (a priori or through training) knowledge of the sequence’s cross-correlation. However, with long PN sequences, the cross-correlation varies from one symbol to another as different parts of the signature sequences are used for different symbols. If the receiver has to calculate the cross-correlation for each symbol, the computational complexity grows exponentially with the number of users.

Multi-user detection methods which do not require knowledge of the sequence’s cross-correlation and whose complexity grows only linearly with the number of users have also been proposed [16], [17]. In [16], users are detected successively; in [17], they are detected simultaneously. In these methods, the receiver reconstructs other users’ transmitted signals by using...
initial decisions about other users' signals. These receivers then use the reconstructed signals to remove CCI from the composite received signals. These methods do not require knowledge of the cross-correlation between the spreading codes. However, there is residual interference due to symbol errors in the initial decisions [18], [19]. Since the performance of the canceller depends on the accuracy of the initial decisions, it is desirable to improve the accuracy of these decisions.

In this paper, we propose a new parallel CCI cancellation technique that utilizes orthogonal convolutional codes [15]. In this method, received signals are both demodulated and decoded. The resulting bit streams are re-encoded and respread, then subtracted from the composite received signals. Owing to the error correcting capability of orthogonal convolutional codes, the residual interference is reduced as the accuracy of the initial decision increases. In other words, the proposed canceller makes one initial decision based on several symbols, consequently the bit-error rate (BER) performance can be improved. Since decoding of the orthogonal convolutional codes increases the processing delay, this system is primarily intended for data communications.

At the same time as the earlier version of this paper was published [20], a similar technique with cascaded CCI cancellation and hard decision decoding of a (7,4) Hamming code was presented [21].

It is of interest to note that a scheme proposed in [22], which combines successive CCI cancellation with orthogonal convolutional coding, has been shown to be capable of achieving Shannon capacity. The implementation complexity of this scheme for successively decoding signals of all users remains a concern, especially at high data rates. Successive cancellation requires signal processing which is significantly faster than that required in parallel cancellation. Therefore, a successive canceller might require more expensive very large scale integration (VLSI) chips which are specially designed to achieve low power consumption and high speed operation. In addition, in successive cancellation, the power of the received signals has to be assigned unequally in order to ensure that users have equitable performance since the CCI is removed successively. Thus, the performance of each user largely depends on the signal-to-interference ratio (SIR). To achieve ideal cancellation, successive cancellation requires very accurate power control.

This paper is organized as follows. In Section II, the conventional CCI canceller is described. In Section III, the new CCI canceller is proposed and its performance is derived. Simulation results and theoretical performance results are shown in Section IV. Section V presents conclusions. For simplicity, we focus on baseband asynchronous CDMA on the AWGN channel.

II. CONVENTIONAL CO-CHANNEL INTERFERENCE CANCELLER

A. System Model

Figure 1 shows a model of an asynchronous CDMA system with $K$ transmitting users. Each user encodes its data using an orthogonal convolutional code. Assume that the code rate is $1/m$ ($M = 2^m$) and the code has constraint length $n_A$. Depending on the output of the convolutional code, one of $M$ orthogonal sequences is chosen by the orthogonal convolutional encoder as one coded symbol [23]. The resulting symbols are then interleaved and modulated with a part of the long PN sequence which is unique for each user and separates one user from another in direct sequence spreading. The processing gain is $G_p = T_s/T_c$, where $T_s$ is the symbol duration and $T_c$ is the chip duration of the PN sequence. The received signal of the $k$th user is

$$S_k(t) = \sqrt{P_k} W^r(t - \tau_k) C_k(t - \tau_k) + n(t) \quad (1)$$

where $W^r(t)$ is one of the orthogonal sequences referred to as the $r$th symbol, $r = 1, \ldots, M$, $C_k(t)$ is the long PN sequence for the $k$th user whose period is much larger than $G_p$ [15], $P_k$ is the signal power for $k$th user (for simplicity, we subsequently assume $P_k = P$ for all $k$), $\tau_k$ is the time delay for the $k$th user ($0 \leq \tau_k \leq T_s$) and $n(t)$ is AWGN with power spectral density $N_0/2$ W/Hz.

Figure 2 depicts a receiver using the conventional CCI canceller with user 1 as the reference user. Here, complete chip synchronization for every user is assumed. In this receiver every user's received signals are first despread and decorrelated with a bank of correlators. Then the most probable orthogonal sequence is selected for each user. Utilizing these initial decisions, the selected sequences for each user are then respread and removed from the composite received signal. After that, user 1's signal is despread, demodulated, de-interleaved and decoded.

B. Performance of the Conventional CCI Canceller

To analyze the performance of the conventional CCI canceller, we assume the users' spreading sequences are random sequences, and that the chip pulse shape is rectangular.
The signal-to-noise ratio (SNR\textsubscript{c1}) after correlation with user 1's sequence at the receiver is given by

\[
\text{SNR}_{c1} = \frac{1}{3Gp^2} \sum_{i=2}^{K} r_{1k} + \frac{N_o}{2E_s}
\]  

(2)

where \( r_{1k} \) is the average interference correlation between sequences of user 1 and user \( k \) and \( E_s = PT_s \) is the energy per symbol [24]. With random spreading sequences, \( r_{1k} \) is equal to \( 2Gp^2 \) [18], [19]. Therefore, the SNR after correlation is

\[
\text{SNR}_{c1} = \frac{1}{3Gp^2} + \frac{N_o}{2E_s}
\]  

(3)

The denominator of (3) corresponds to approximating the interference as Gaussian noise. The error probability of the initial decision conditioned on the symbol taking the \( r \)th sequence is the probability that the \( r \)th sequence correlator output \( U_r \) is smaller than at least one of the other sequences' outputs \( U_i \) \((i = 1, \cdots, M, i \neq r)\) and is given by [18], [19], and [25]

\[
P_{e_r} = 1 - \int_{-\infty}^{\infty} P(U_1 < U_r, U_2 < U_r, \cdots, U_M < U_r) \cdot p(U_r) \, dU_r
\]

\[
= 1 - \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \exp \left( -\frac{u^2}{2} \right) \cdot \{(1 - Q(u + \sqrt{\text{SNR}_{c1}}))^{M-1} \, du
\]

\[
\leq (M-1) \cdot Q(\sqrt{\text{SNR}_{c1}})
\]  

(4)

where \( Q(t) \) is the standard Gaussian upper cumulative distribution function

\[
Q(t) = \frac{1}{\sqrt{2\pi}} \int_{t}^{\infty} \exp \left( -\frac{x^2}{2} \right) \, dx.
\]  

(5)

The validity of the Gaussian approximation for the interference as used in (4) will be shown later through simulation. When an error in the initial decision occurs, the CCI cancellation process actually doubles the interference power since the canceller adds another interference signal instead of canceling it. Suppose that the \( n \)th symbol of user 1 overlaps with \( n - 1 \)th and \( n \)th symbol of user \( k \) for the duration of \( T' \) and \( T_s - T' \), respectively. We assume that errors occur in adjacent symbols of the \( n \)th user independently with probability \( P_{e_n} \).

The average interference from symbol \( n - 1 \) of user \( k \) on symbol \( n \) of user 1 is

\[
2P_{e_n} \left( \frac{T'}{T_s} \right) \frac{2}{3Gp}.
\]  

(6)

Similarly, the average interference from symbol \( n \) of user \( k \) is

\[
2P_{e_n} \left( \frac{T_s - T'}{T_s} \right) \frac{2}{3Gp}.
\]  

(7)

From (6) and (7), the average total interference on symbol \( n \) of user 1 from symbols \( n - 1 \) and \( n \) of user \( k \) is

\[
2P_{e_n} \left( \frac{T'}{T_s} + \frac{T_s - T'}{T_s} \right) \frac{2}{3Gp} = 2P_{e_n} \frac{2}{3Gp}.
\]  

(8)

There are \( K - 1 \) interfering users. Thus, the SNR after CCI cancellation is

\[
\text{SNR}_{c2} = \frac{1}{2P_{e_n} \frac{2}{3Gp} + \frac{N_o}{2E_s}}
\]  

(9)

From (9), it is clear that the SNR after cancellation is dependent on the error probability of the initial decision. Here, we assume that the receiver can accurately estimate the power of each received signal. An upper bound on the bit error
where $P_e(d)$ is the probability of selecting the incorrect path [23]

$$P_e(d) = Q\left(\sqrt{\frac{d \cdot \text{SNR}_{e2}}{2}}\right)$$

(11)

$B_d$ is the total number of nonzero information bits on all paths of weight $d$ [23], [26], and $d_{\text{free}}$ is the minimum free distance. If the weight spectrum $B_d$ is known, the upper bound in (10) can be calculated. However, the bound becomes looser as the BER becomes worse. Quite often, the first few terms in (10) can be used to approximate the performance of the canceller. We use the first four terms in approximating the performance of both the conventional and the new CCI cancellation schemes.

III. NEW CCI CANCELLER

A. System Model

Now we propose a new parallel CCI canceller which uses an orthogonal convolutional code. As discussed above, the performance of the CCI canceller depends on the initial decision. To reduce the error probability of the initial decision, our proposed system utilizes the error correcting capability of orthogonal convolutional codes.

A block diagram of the proposed canceller is given in Fig. 3. Following the maximum likelihood decisions on each user’s sequence, the proposed canceller de-interleaves coded symbols and decodes the orthogonal convolutional codes using a soft decision Viterbi decoder. The decoded data are then re-encoded, assigned an orthogonal sequence, interleaved, and re-spread by the long PN sequence. During this period, the received signal is also in memory whose size equals the interleaving size. As in the conventional canceller, after re-spread the signals of users $2 \sim K$ are removed from the composite received signal in memory, and user 1’s signal is despread, decorrelated, de-interleaved, and decoded.

Note that the Viterbi decoders required in the proposed scheme are already present in the conventional CCI cancellation schemes in which each user has a Viterbi decoder. If the required delay is not acceptable (recall that we are considering transmission of data which may tolerate a relatively long delay and, quite often, the receiver may not have to process consecutive data packets) with reuse of the same Viterbi decoder after CCI cancellation, the delay can be reduced by employing two Viterbi decoders per user. Later we will show that, for the same performance, the complexity of each decoder may be reduced by more than a factor of two so that the overall complexity is still reduced.

B. Performance of the Proposed CCI Cancellers

In evaluating the performance of the proposed CCI canceller, we use the same assumptions as those for the conventional canceller presented in Section II-B. The number of symbol errors caused by choosing a wrong path after re-encoding is the same as the distance of the wrong path from the correct path in the state diagram of the convolutional code. There are $A_d$ paths which cause $d$ symbol errors after re-encoding and the probability with which the Viterbi decoder chooses a wrong path of the distance $d$ is

$$P_{e1}(d) = Q\left(\sqrt{\frac{d \cdot \text{SNR}_{n1}}{2}}\right)$$

(12)

where

$$\text{SNR}_{n1} = \frac{1}{2(2K - 1)} \frac{N_o}{3Gp} + \frac{N_o}{2E_s}.$$  

(13)

The average symbol error probability caused by one user after re-encoding is then

$$P_{e_n} \approx \sum_{d=d_{\text{free}}}^{d_{\text{max}}+3} d \cdot A_d \cdot P_{e1}(d).$$

(14)

Therefore, the SNR after cancellation is

$$\text{SNR}_{n2} = \frac{1}{2P_{e_n} - \frac{2(2K - 1)}{3Gp} \frac{N_o}{2E_s}}.$$  

(15)

From SNR$_{n2}$, the approximate error probability after cancellation is

$$P_{n2}(d) = Q\left(\sqrt{\frac{d \cdot \text{SNR}_{n2}}{2}}\right).$$  

(17)

C. Performance Improvement

The improvement in performance of the proposed canceller over the conventional canceller is the difference between SNR$_{n2}$ and SNR$_{e2}$ in (9) and (15). This difference, which is due to the error correction capability of the orthogonal convolutional codes, is

$$\text{SNR}_{n2} - \text{SNR}_{e2} = \frac{1}{2P_{e_n} - \frac{2(2K - 1)}{3Gp} \frac{N_o}{2E_s}} - \frac{1}{2P_{e_c} - \frac{2(2K - 1)}{3Gp} \frac{N_o}{2E_s}} - \frac{2\alpha(P_{e_c} - P_{e_n})}{(2\alpha P_{e_n} + \beta)(2\alpha P_{e_c} + \beta)}$$

(18)
TABLE 1

<table>
<thead>
<tr>
<th>Constraint Length</th>
<th>Generator Polynomials (Octal)</th>
<th>$A_{d_{free}} + d$</th>
<th>$B_{d_{free}} + d$</th>
</tr>
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<td>3</td>
<td>4, 5, 6</td>
<td>1, 1, 2, 2</td>
<td>1, 2, 5, 10</td>
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<tr>
<td>4-ary</td>
<td>12, 15</td>
<td>1, 5, 5, 19</td>
<td>1, 14, 21, 94</td>
</tr>
<tr>
<td>8-ary</td>
<td>11, 13, 15</td>
<td>1, 1, 3, 5</td>
<td>1, 2, 7, 13</td>
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<tr>
<td>16-ary</td>
<td>11, 13, 14, 15</td>
<td>1, 1, 2, 4</td>
<td>1, 2, 5, 12</td>
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<tr>
<td>5</td>
<td>23, 33, 26</td>
<td>1, 2, 3, 7</td>
<td>1, 5, 8, 25</td>
</tr>
<tr>
<td>6</td>
<td>45, 67, 56</td>
<td>1, 2, 3, 9</td>
<td>1, 5, 7, 34</td>
</tr>
<tr>
<td>7</td>
<td>114, 156, 133</td>
<td>1, 2, 3, 13</td>
<td>1, 4, 8, 66</td>
</tr>
<tr>
<td>9</td>
<td>557, 663, 711</td>
<td>1, 2, 7, 12</td>
<td>1, 5, 22, 54</td>
</tr>
</tbody>
</table>

Ex. Constraint Length 4
8-ary Optimum Code

where

$$\alpha = \frac{2(K - 1)}{3Gp}$$ (19)

$$\beta = \frac{N_o}{2E_s}$$ (20)

From (4) and (14),

$$P_{e_c} - P_{e_n} \approx (M - 1) \cdot Q(\sqrt{\text{SNR}}) - \sum_{d = d_{free}}^{d_{free} + 3} d \cdot A_d P_{n1}(d)$$

$$= (M - 1) \cdot Q(\sqrt{\text{SNR}}) - \sum_{d = d_{free}}^{d_{free} + 3} d \cdot A_d Q\left(\sqrt{\frac{d \cdot \text{SNR}}{2}}\right)$$ (21)

where

$$\text{SNR} = \frac{1}{\alpha + \beta}.$$ (22)

If the SNR is large, the first term of (14) dominates $P_{e_n}$. That is

$$P_{e_c} - P_{e_n} \approx (M - 1) \cdot Q(\sqrt{\text{SNR}}) - d_{free} \cdot A_{d_{free}} \cdot Q(\sqrt{\frac{d_{free} \cdot \text{SNR}}{2}}).$$ (23)

When $P_{e_c} - P_{e_n} \geq 0$, the proposed canceller performs better than the conventional canceller. In this instance

$$M - 1 \cdot d_{free} \cdot A_{d_{free}} \geq \frac{Q\left(\sqrt{\frac{d_{free} \cdot \text{SNR}}{2}}\right)}{Q(\sqrt{\text{SNR}})}.$$ (24)

As the constraint length of the orthogonal convolutional codes increases, $d_{free}$ increases, and the numerator on the right side of (24) decreases exponentially. Thus, the proposed canceller shows performance improvement at lower SNR with orthogonal convolutional codes of larger constraint length. This improvement can also be seen in (18) where the numerator increases and the denominator decreases with increasing constraint lengths of the orthogonal convolutional code.

IV. RESULTS

We assume that each time delay $\tau_i$ ($i = 1, 2, \cdots, K$) is known perfectly [18], [19]. We also assume that the processing gain $Gp$ is 128 and the code used is the eight-ary orthogonal convolutional code with optimum generator polynomials given in Table I [15], [23]. This code has rate $1/3$.

Figure 4 shows the BER performance of the orthogonal convolutional code on the AWGN channel. In this figure, the points are simulation results and the lines indicate theoreti-
Fig. 4. Performance of orthogonal convolutional code, number of users = 1, code rate = 1/3, and constraint length = 4.

Fig. 5. BER versus $E_b/N_0$, number of users = 20, code rate = 1/3, and constraint length = 4.

Fig. 6. BER versus number of users, $E_b/N_0 = 5$ dB, code rate = 1/3, and constraint length = 4.

Theoretical performance calculated from the first four terms of the Gaussian approximation in (10). Results for constraint lengths four, seven, and nine are shown. Due to the agreement of the simulated points and theoretical values with $d = d_{\text{free}} - d_{\text{free}} + 3$, we conclude that the approximation is appropriate.

Figures 5 and 6 compare simulation results with theoretical performance for three schemes: with no CCI canceller, with the conventional canceller, and with the proposed canceller. During simulation we assumed asynchronous CDMA and eight samples per chip so that $\tau_k$ was uniform on $[0, T_c]$ with quantization $T_c/8$. An interleaver with a size of 12 symbols x 14 symbols was employed to ensure that symbol errors were independent. The performance without a canceller is calculated using (10) and (11) with $\text{SNR}_{c2}$ replacing $\text{SNR}_{c2}$. Since the simulated points are close to the theoretical values, once again we conclude that the approximations introduced in Section II are acceptable.

In Figs. 7–10, theoretical performances without a canceller, with the conventional canceller, and with the proposed canceller are presented. Fig. 7 shows BER versus $E_b/N_0$ for a code with constraint length of nine. When $K = 20$ (the number of users which transmit signals at the same time), the performance of the proposed canceller is very close to the performance with $K = 1$ and superior to the conventional canceller by 1 dB at BER = $10^{-3}$ and 1.5 dB at BER = $10^{-5}$. The superiority of the proposed system becomes more pronounced as the number of users increases. For example, using a constraint length nine code, the difference is more than 10 dB for $K = 60$ at BER = $10^{-5}$.

Figure 8 shows BER versus number of users. When compared to the conventional canceller, it is evident from this figure that the number of simultaneous users supported by the proposed canceller with BER = $10^{-3}$ increases by a factor of 2 $\sim$ 3 when $E_b/N_0$ is 5 dB, and by a factor of 1.5 $\sim$ 2 when $E_b/N_0$ is 10 dB. It is also clear that with up to about 40 users, the performance of the proposed CCI canceller is near the performance without interference.

Figure 9 shows BER versus constraint length of the convolutional code. From this figure, it is clear that even though the conventional canceller does not work well when $K = 60$, the proposed canceller works quite effectively. Also, the performance of the proposed canceller improves with increasing constraint lengths, especially when $K$ is large.

The proposed cancellation scheme can also be viewed as an effective way of reducing system implementation complexity.
size of the path memory is reduced as the required decoding depth becomes smaller [27]. Recall that the number of Viterbi decoders can be doubled in order to shorten the delay. Even with this taken into account, the overall complexity is reduced by a factor of two.

Figure 10 shows BER versus code rate with the constraint length four. The generator polynomials of codes with rate 1/2, 1/3, and 1/4 are given in Table I. The performance of the proposed canceller improves as the code rate is reduced since it takes full advantage of the superior distance structure of a lower rate code. For the conventional canceller, it is interesting to note that the optimum code rate is 1/3. This is because, from (4), the error probability of the initial decision increases as $M$ increases and the conventional canceller does not perform error correction. Therefore, even though a larger code has a better distance structure, the conventional canceller makes more symbol errors and the performance becomes worse when code rate is less than 1/3. The performance with the proposed canceller improves because the error probability of the initial decision is lowered with better codes.

V. CONCLUSION

In this paper, we have proposed a new parallel CCI cancellation technique which improves the accuracy of the initial decision by utilizing the error correction capability of orthogonal convolutional codes. The performance of the proposed canceller has been computed and compared with the conventional canceller. It is shown that the proposed canceller offers $1.5 \sim 3$ times higher user capacity than the conventional canceller. Given a processing gain of 128, the proposed CCI canceller cancels nearly all multi-user interference for up to about 40 users. We have also shown that the canceller can significantly benefit from the use of a code with large
constraint length. For the same performance, the proposed canceller can effectively reduce the decoding complexity. Finally, with lower rate codes, the performance of the proposed canceller improves while the performance of the conventional canceller may not.

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