A Novel Cumulant Based MUSIC Like DOA Estimation Algorithm with Multicarrier Modulation*

Yukitoshi SANADA†, Junichi TAKADA‡†, and Kiyomichi ARAKI†, Members

SUMMARY A novel cumulant based MUSIC like DOA estimation algorithm for multicarrier modulation has been proposed in this paper. While the conventional MUSIC algorithm is not applicable to a correlation matrix calculated from received signals transmitted over the different carriers, the proposed algorithm can estimate the DOA of the signals with multicarrier modulation. The proposed algorithm does not require the sensor array responses for the frequency range of the interest and the initial phases of the carriers. With the proposed algorithm the number of signals whose DOA are estimated can be increased and the accuracy of the DOA estimation can be improved by employing larger number of carriers.

key words: DOA estimation, MUSIC, cumulant, multicarrier

1. Introduction

As wireless personal communications have become popular and popular, the demand for accommodating larger number of users arises. One of the main problems which limit the number of users in cellular systems is co-channel interference (CCI). To avoid CCI, multiple access techniques such as FDMA, TDMA, or CDMA, utilize the orthogonality of signals over frequency, time, or spreading sequences. In addition to these multiple access techniques, a new multiple access technique called Space Division Multiple Access (SDMA) has been investigated [1],[2]. In this multiple access technique, the direction-of-arrival (DOA) estimation of the signals from mobile users is essential. A number of DOA estimation algorithms have been proposed during the last decade [3]–[5]. Among the proposed algorithms, the MUSIC algorithm is the most popular subspace method due to its applicability to arrays of arbitrary orientation and response [4]. In [6], cumulants of received signals are utilized to estimate array calibration and DOA of signals.

Not only the CCI, but also intersymbol interference (ISI) due to multipath fading reduces the capacity of the cellular systems. Multipath fading deteriorates the quality of communication links and makes high rate and high quality data transmission difficult. To improve the quality of communication links, multicarrier modulation techniques have been considered [7]. In multicarrier modulation, information bits are transmitted through multiple carriers in parallel. Owing to the longer bit duration, the amount of ISI decreases. In addition, frequency diversity among the carriers can be achieved because of the frequency selectivity of the channel fading.

In this paper, a novel cumulant based MUSIC like DOA estimation algorithm which is designed for multicarrier modulation is proposed. Although many literatures like [8]–[13] have been concerning on the DOA estimation problem of wideband signals, none of them has considered multicarrier modulation. In those techniques, the sensor array responses must be known for the frequency range of the interest. This requirement is troublesome for wireless communication systems. In addition, the initial phases of the carriers are not available to a receiver and this uncertainty prevents the conventional MUSIC based algorithms to be applied. On the other hand, the proposed algorithm does not require the knowledge of the initial phases of the carriers. It only requires the condition that the signals from the different mobiles are uncorrelated. This assumption is easy to achieve for the wireless communication systems with PSK modulation schemes as the mobiles transmit different information bits.

The proposed algorithm estimates only the DOA of the signals. Each signal from a mobile consists of multiple elementary waves that are from almost the same direction. These elementary waves have different Doppler shifts and create different phase shifts between the carriers. It is possible to separate those elementary waves, the delay as well as the DOA of the waves can be estimated by comparing the phase shifts of the waves on all the carriers [14]. However, to estimate the delay of each elementary wave, the number of carriers must be so large. This is beyond the scope of the proposed algorithm. It is here assumed that those elementary waves create one signal and the proposed algorithm estimates the DOA of the composite signal.
This paper is organized as follows. In Sect. 2, the signal model is described. In Sect. 3, the proposed algorithm is presented. Simulation results are shown in Sect. 4. Section 5 presents our conclusions.

2. Signal Model

It is assumed that a base station employs an antenna array with $K$ elements, which are in an equi-spaced, linear configuration. The number of carriers is $M$ and the signal from a mobile on each carrier is modulated by BPSK. There are $L$ mobiles assigned to the base station. The received signal after demodulation on the carrier $m$ of the antenna $k$ is represented as

$$ r_{km}(t) = \sum_{\ell=1}^{L} S_{m\ell}(t) \exp(-j \frac{2\pi}{\lambda_m} (k-1) \Delta \sin(\theta_{\ell})) + n_{km}(t) $$  \hspace{1cm} (1)

where $\lambda_m$ is the wavelength of the $m$-th carrier, $\Delta$ is the distance between the adjacent elements, $n_{km}(t)$ is the additive white Gaussian noise (AWGN) with zero mean and covariance $\sigma^2$, and $S_{m\ell}(t)$ is the signal on the $m$-th carrier from the $\ell$-th mobile, which is given by

$$ S_{m\ell}(t) = g_{m\ell}(t) \sqrt{P_{m\ell}} b_{\ell}(t) \exp(-j (\varphi_{m\ell}(t) + \phi_{m\ell})) $$  \hspace{1cm} (2)

where $g_{m\ell}(t)$ and $\varphi_{m\ell}(t)$ account for the attenuation and the phase shift of the channel, respectively. $P_{m\ell}$ and $\phi_{m\ell}$ are the power and the phase shift of the signal on the $m$-th carrier from the $\ell$-th mobile. $b_{\ell}$ is the information bits of the $\ell$-th mobile. It is assumed that in order to realize frequency diversity gain, the same information bits are transmitted on all the carriers at the same time [15].

From these assumptions, the correlation between the received signals is given by

$$ E[S_{m\ell}(t)S_{m'\ell'}^*(t)] = \begin{cases} C_{m\ell,m'\ell'} & \ell \neq \ell' \\ 0 & \ell = \ell' \end{cases} $$  \hspace{1cm} (3)

where $^*$ denotes complex conjugate and $C_{m\ell,m'\ell'}$ is the correlation between the signals on the $m$-th carrier from the $\ell$-th mobile and on the $m'$-th carrier from the $\ell'$-th mobile. $C_{m\ell,m'\ell'}$ is not able to be estimated at the receiver for each signal when multiple signals from different mobiles are received at the same time. This is because the phase shifts, $\{\phi_{m\ell}\}$ and $\{\varphi_{m\ell}\}$, are independent between the carriers and between the signals from the different mobiles.

The conventional MUSIC algorithm extracts the correlation values (elements of the steering vector) of the signals which are from the same mobile and received at the different antennas. It then converts the correlation values to the DOA of the signal with the information about the locations of the antennas. If the signals are transmitted on the different carriers, the correlation values contain the phase shifts, $\{\phi_{m\ell}\}$ and $\{\varphi_{m\ell}\}$, as well as the DOA information. Those phase shifts are independent between the carriers and unknown to the receiver. It is also hard to calculate them separately when multiple mobiles transmit their signals at the same time. Therefore, the conventional MUSIC algorithm cannot estimate the DOA of the signals over the different carriers. Thus, the number of signals whose directions can be estimated is in proportion to $K - 1$.

On the other hand, the proposed algorithm chooses a reference signal to calculate the cumulants of the received signals. The reason why the proposed algorithm utilizes the cumulants is that they can preserve the phase relations between the signals over the different carriers. The proposed algorithm selects the signals to make the calculated cumulants contain the same amount of those phase shifts relative to the reference signal. Thus, it is possible to eliminate the influence of the phase shifts and convert the cumulants to the DOA information. The number of signals whose directions can be estimated is extended to the order of $MK$ with the proposed algorithm by using the signals over the different carriers.

3. Proposed Cumulant Based Algorithm

3.1 Properties of Cumulants

Higher order spectra defined in terms of higher order statistics (“cumulants”) of a signal is a useful tool as it reveals not only amplitude information about the process, but also phase information (Derivation of cumulants is given in Appendix) [16], [17].

Followings are the some important properties of cumulants.

- If $\alpha_{i1}, \ldots, h_i$, are constants, and $\mu_{i1}, \ldots, h_{i}$, are random variables, then
  $$ \text{cum}(\alpha_{i1}\mu_{11}, \ldots, \alpha_{ih}\mu_{h}) = \left( \prod_{i=1}^{h} \alpha_{i} \right) \text{cum}(\mu_{11}, \ldots, \mu_{h}). $$  \hspace{1cm} (4)

- If the random variables $\{\mu_i\}, i = 1, 2, \ldots, h$, are independent of the random variables $\{\nu_i\}, i = 1, 2, \ldots, h$, then
  $$ \text{cum}(\mu_1 + \nu_1, \ldots, \mu_h + \nu_h) = \text{cum}(\mu_1, \ldots, \mu_h) + \text{cum}(\nu_1, \ldots, \nu_h). $$  \hspace{1cm} (5)

- If a subset of the $h$ random variables $\{\mu_i\}$ is independent of the rest, then
  $$ \text{cum}(\mu_{11}, \ldots, \mu_{h}) = 0. $$  \hspace{1cm} (6)

- Cumulants can suppress Gaussian noise and draw non-Gaussian signal out of Gaussian noise.
3.2 Proposed MUSIC Like Algorithm

The proposed MUSIC like DOA estimation algorithm utilizes the cumulants of the received signals over the different carriers and antennas. Now it is assumed that the differences of the carrier frequencies are small as compared with the carrier frequencies themselves. Thus,

\[
\frac{2\pi}{\lambda_m} \Delta \sin(\theta_c) \simeq \frac{2\pi}{\lambda} \Delta \sin(\theta_c) = \psi(\theta_c). \tag{7}
\]

Let \( r_{k,m_c}(t) \) be a reference signal, which is received on the \( m_c \)-th carrier of the \( k_c \)-th antenna, in order to calculate the cumulants of the received signals. The proposed algorithm requires to calculate the correlation between the reference signal and the other received signals through cumulants. The cumulants are then given as

\[
x(k_c,m_c; k_1, k_2, \ldots, k_N) = \text{cum}(r_{k,m_c}(t), r_{k_1}^*(t), r_{k_2}(t), \ldots, r_{k,N}^*(t)), \tag{8}
\]

where \( k_c \) and \( k_1, k_2, \ldots, k_N \) represent the index of the antennas. Here, \( k_c \) and \( k_1, \ldots, k_N \) can be from 1 to \( K \) while \( m_c \) is one of \( M \) carriers. In Eq. (8), \( \{S_{mc}(t)\} \) are independent for different \( \ell \). Thus,

\[
x(k_c,m_c; k_1, k_2, \ldots, k_N) = \sum_{\ell=1}^{L} \text{cum}(S_{mc}(t), \exp(-j(k_c - 1)\psi(\theta_c)), S_{1\ell}(t), \exp(j(k_2 - 1)\psi(\theta_c)), \ldots, S_{N\ell}(t), \exp(j(k_N - 1)\psi(\theta_c)))
\]

\[
= \sum_{\ell=1}^{L} \left[ \exp(-jNk_c - \sum_{n=1}^{N} k_n) \psi(\theta_c) \right]
\]

\[
\text{cum}(S_{mc}(t), S_{1\ell}^*(t), \ldots, S_{N\ell}(t), S_{N\ell}^*(t)). \tag{9}
\]

Here, it is assumed that the AWGN is suppressed due to the property of the cumulants. In Eq. (9), by taking the combinations of \( N, k_c, \) and \( k_n \), the cumulants with different values of the term, \( Nk_c - \sum_{n=1}^{N} k_n \), can be calculated. In the term, \( N \) is a predetermined value and \( k_c \) and \( k_n \) can be selected from 1 to \( K \). Let \( \rho_{i} \), \( i = 1, \ldots, D \), be the resultant combinations of the term and \( x_i \) denote the corresponding \( x \), i.e.

\[
x_i(k_c,m_c; k_1, k_2, \ldots, k_N) = \text{cum}(S_{mc}(t), S_{1\ell}^*(t), \ldots, S_{N\ell}(t), S_{N\ell}^*(t)). \tag{10}
\]

where

\[
\rho_1 = \min(Nk_c - \sum_{n=1}^{N} k_n), \tag{11}
\]

\[
\rho_2 = \min(Nk_c - \sum_{n=1}^{N} k_n) + 1, \tag{12}
\]

\[
\vdots
\]

\[
\rho_D = \max(Nk_c - \sum_{n=1}^{N} k_n). \tag{13}
\]

Here, "\( \min \)" represents minimization of the term \( Nk_c - \sum_{n=1}^{N} k_n \) by changing the values of \( k_c \) and \( k_n \) and "\( \max \)" implies the maximization of the term. From Eq. (10), the following vector can be defined as

\[
X = [x_1, x_2, \ldots, x_D]. \tag{14}
\]

The correlation matrix, \( R \), is defined as

\[
R = XX^\dagger = AA^\dagger \tag{15}
\]

where \( ^\dagger \) represents the conjugate transpose and

\[
A = [a(\theta_1), \ldots, a(\theta_L)], \tag{16}
\]

\[
a(\theta_\ell) = [\exp(-j\rho_1 \psi(\theta_\ell)), \exp(-j\rho_2 \psi(\theta_\ell)), \ldots, \exp(-j\rho_D \psi(\theta_\ell))]^T, \tag{17}
\]

\[
R_s = ss^\dagger \tag{18}
\]

\[
\hat{s} = \left[ \begin{array}{c}
\text{cum}(S_{m_c}(t), S_{1\ell}(t), \ldots, S_{N\ell}(t), S_{N\ell}^*(t)) \\
\text{cum}(S_{m_c}(t), S_{1\ell}^*(t), \ldots, S_{N\ell}(t), S_{N\ell}^*(t)) \\
\vdots \\
\text{cum}(S_{m_c}(t), S_{1\ell}^*(t), \ldots, S_{N\ell}(t), S_{N\ell}^*(t))
\end{array} \right]. \tag{19}
\]

The matrix, \( R_s \), consists of the product of the correlation between the carriers. The condition required to calculate the matrix is that the same information bit is transmitted over the multiple carriers as mentioned in Sect. 2. The rank of the correlation matrix, \( R_s \), is 1. Therefore, the directions of the signals can be estimated through this correlation matrix only when the number of signals is one. In order to increase the rank of the correlation matrix, forward/backward smoothing technique must be employed[18]. This is possible when the differences between the adjacent elements of \( \{\rho_i\} \) in Eqs. (11) to (13) have the same value. In other words, \( k_c \) and \( k_n \) are selected to make the differences same. From Eq. (14), the subspace of the received signal vector is

\[
X_d^f = [x_d, x_{d+1}, \ldots, x_{d+D_{\text{sub}}-1}] \tag{20}
\]

where \( D_{\text{sub}} \) is the size of the correlation matrix after the averaging. Then, the covariance matrix of the \( d \)-th subspace is given by

\[
R_d^f = \frac{1}{D} \sum_{d=1}^{D} R_d \tag{21}
\]
where

\[ R_d^f = X_d^f X_d^{f \dagger}. \]

(22)

For the backward smoothing the subspace of the vector is defined as

\[ X_d^b = [x_{d-D-d+1}, x_{D-d}^*, \ldots, x_{D-D_{sub}-d+2}^*] \]

(23)

and the covariance matrix for the backward smoothing is

\[ R_d^b = \frac{1}{D} \sum_{d=1}^{D} R_d^b \]

(24)

where

\[ R_d^b = X_d^b X_d^{b \dagger}. \]

(25)

From (24) and (25) the correlation matrix after averaging is given by

\[ R_{\alpha \alpha} = \frac{R_f^f + R_b^b}{2} = \sum_{l=1}^{L} \lambda_l e_l e_l^\dagger + \sigma^2 \sum_{l=L+1}^{D_{sub}} e_l e_l^\dagger \]

(26)

where \( \lambda_l \) is the \( l-th \) eigenvalue of the correlation matrix, \( e_l \) is the corresponding eigenvector, and \( \sigma^2 \) is the residual power of the noise which can not be eliminated through the calculation of the cumulants. Let \( E_n \) be defined as

\[ E_n = [e_{L+1}, \ldots, e_{D_{sub}}]. \]

(27)

The peaks of the function,

\[ P_{\mu \nu}(\theta_i) = 1/|E_n^\mu \alpha(\theta_i)|^2, \]

(28)

provide the DOA estimates.

3.3 Direction Finding Capability

For the conventional MUSIC algorithm the maximum number of signals whose directions can be estimated is equal to (the number of sensors-1) as it can not utilize the received signals over the different carriers. On the other hand, the proposed algorithm is able to employ all the signals transmitted on different carriers. If the forward/backward smoothing is applicable, the maximum number of the signals whose directions can be estimated is equal to \( 2D/3 \)[18]. The following conditions have to be satisfied in terms of \( \rho_i \)[19].

- For the forward smoothing virtual subarrays defined in Eq. (20) have the same form, i.e., the differences between the adjacent elements of the vector in Eq. (20), \( X_d^f \), must have the identical phase shift pattern for any index \( d \).

- For the backward smoothing the vector in Eq. (23), \( X_d^b \), must have centro-symmetry as defined by: \( x_d - x_1 = x_d - x_{D-d+1} \), for \( d = 1, \ldots, D \).

From Eq. (9), \( \rho_i \) can take the values from \(-N(K-1)\) to \(+N(K-1)\). Thus, the size of the correlation matrix in Eq. (15), \( D \), is at most \( 2N(K-1) + 1 \) and the maximum number of the signals whose directions can be estimated is \( 2(2N(K-1) + 1)/3 \).

4. Simulation Results

4.1 Simulation Conditions

Simulation conditions are shown in Table 1. The proposed algorithm is simulated on an AWGN channel and a Rician fading channel. On the Rician fading channel, Rician parameter is set to 4. It is also assumed that there are 2 antennas which are in \( \lambda/2 \) distance apart. The power of the signal, \( P_{m \ell} \), is set to 1 for any number of \( m \) and \( \ell \). The number of carriers assumed is 2, 8, and 16. When there are 2 carriers, the following 8 different \( \{x_i\} \) are calculated.

\[ x_1(11; 2, 2) = \text{cum}(r_{11}(t), r_{21}^*(t), r_{11}(t), r_{22}^*(t)) = \sum_{\ell=1}^{2} e^{-j(2\times1-2-2)t} \psi(\theta_i) \text{cum}(S_{1\ell}(t), S_{1\ell}^*(t), S_{2\ell}(t), S_{2\ell}^*(t)), \]

(29)

\[ x_2(11; 1, 2) = \text{cum}(r_{11}(t), r_{11}^*(t), r_{11}(t), r_{22}^*(t)) = \sum_{\ell=1}^{2} e^{-j(2\times1-1-2)t} \psi(\theta_i) \text{cum}(S_{1\ell}(t), S_{1\ell}^*(t), S_{1\ell}(t), S_{2\ell}^*(t)), \]

(30)

\[ x_2(11; 2, 1) = \text{cum}(r_{11}(t), r_{21}^*(t), r_{11}(t), r_{12}^*(t)) = \sum_{\ell=1}^{2} e^{-j(2\times1-2-1)t} \psi(\theta_i) \]

Table 1

<table>
<thead>
<tr>
<th>Simulation conditions</th>
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<tbody>
<tr>
<td>Number of Antennas</td>
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<tr>
<td>Antenna Separation</td>
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<tr>
<td>Number of Carriers</td>
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<tr>
<td>Carrier Frequency of the m-th Carrier</td>
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<tr>
<td>Number of Signals</td>
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<tr>
<td>DOA</td>
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<tr>
<td>Modulation</td>
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<tr>
<td>Bit Rate</td>
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<tr>
<td>Sample Rate</td>
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<tr>
<td>Speed of Vehicle</td>
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<tr>
<td>Rician Parameter</td>
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</table>
\[ x_3(11; 1, 1) = \sum_{\ell=1}^{2} e^{-j(2\times1-1-1)\psi(\theta_{\ell})} \]

\[ \text{cum}(S_{1\ell}(t), S_{1\ell}^*(t), S_{1\ell}(t), S_{1\ell}^*(t)) \] (31)

\[ x_3(21; 2, 2) = \sum_{\ell=1}^{2} e^{-j(2\times2-2-2)\psi(\theta_{\ell})} \]

\[ \text{cum}(S_{1\ell}(t), S_{1\ell}^*(t), S_{1\ell}(t), S_{1\ell}^*(t)) \] (32)

\[ x_4(21; 1, 2) = \sum_{\ell=1}^{2} e^{-j(2\times2-2-1-1)\psi(\theta_{\ell})} \]

\[ \text{cum}(S_{2\ell}(t), S_{2\ell}^*(t), S_{2\ell}(t), S_{2\ell}^*(t)) \] (33)

\[ x_4(21; 2, 1) = \sum_{\ell=1}^{2} e^{-j(2\times2-2-2)\psi(\theta_{\ell})} \]

\[ \text{cum}(S_{2\ell}(t), S_{2\ell}^*(t), S_{2\ell}(t), S_{2\ell}^*(t)) \] (34)

\[ x_5(21; 1, 1) = \sum_{\ell=1}^{2} e^{-j(2\times2-1-1)\psi(\theta_{\ell})} \]

\[ \text{cum}(S_{3\ell}(t), S_{3\ell}^*(t), S_{3\ell}(t), S_{3\ell}^*(t)) \] (35)

\[ x_5(21; 2, 1) = \sum_{\ell=1}^{2} e^{-j(2\times2-2-1)\psi(\theta_{\ell})} \]

\[ \text{cum}(S_{3\ell}(t), S_{3\ell}^*(t), S_{3\ell}(t), S_{3\ell}^*(t)) \] (36)

After combining Eqs. (30) and (31), (32) and (33), (34) and (35) to calculate \( x_k \), the covariance matrix, \( R \), is calculated as follows.

\[ R = AR_eA^H \] (37)

where

\[ A = \begin{bmatrix} e^{-j(-2)\psi(\theta_1)} & e^{-j(-2)\psi(\theta_2)} \\ e^{-j(-1)\psi(\theta_1)} & e^{-j(-1)\psi(\theta_2)} \\ e^{-j(0)\psi(\theta_1)} & e^{-j(0)\psi(\theta_2)} \\ e^{-j(1)\psi(\theta_1)} & e^{-j(1)\psi(\theta_2)} \\ e^{-j(2)\psi(\theta_1)} & e^{-j(2)\psi(\theta_2)} \end{bmatrix} \] (38)

\[ R_e = ss^H \] (39)

\[ s = \begin{bmatrix} \text{cum}(S_{11}(t), S_{11}^*(t), S_{11}(t), S_{11}^*(t)) \\ \text{cum}(S_{12}(t), S_{12}^*(t), S_{12}(t), S_{12}^*(t)) \end{bmatrix} \] (40)

The forward/backward averaging is carried out on the covariance matrix, \( R \), and the peaks of the function given in Eq. (28) is calculated. The maximum number of the signals whose directions can be estimated is 3.

When the number of carriers is 8, 7 correlation matrices corresponding to the sets of signals as shown in Fig. 1 are calculated. For the DOA estimation those correlation matrices are averaged in order to eliminate the noise. Similar to the system with 8 carriers, 15 correlation matrices are calculated for averaging when there are 16 carriers.

The array data for the conventional MUSIC algorithm are also generated under the conditions of Table 1. For the comparison the same array data as the proposed algorithm are utilized to calculate the DOA of the signals. This means that the signals from the different carriers are treated as the different data (for example, when there are 2 carriers and 2 antennas, 4 array data are obtained). When there are 2 antennas and 8 carriers, 16 signals are available. Therefore, the correlation matrix with the size of 16 by 16 is calculated and the conventional MUSIC algorithm is applied. The correlation matrix with the size of 32 by 32 is also calculated when there are 16 carriers.

4.2 DOA Estimation Performance

The direction finding capabilities with the conventional MUSIC algorithm and the proposed algorithm on the AWGN channel are shown in Figs. from 2 to 4. \( Eb/N_0 \) is set to 10 [dB]. The conventional MUSIC algorithm is applied to the received signals over 2 different carri-
ers and 2 different antennas. As shown in those figures, it is not possible to find out the DOA of the signals with the conventional MUSIC algorithm. On the other hand, through the same received signals, the DOA of the 3 signals can be identified with the proposed algorithm. When the number of samples/carrier used to calculate the correlation matrix is 1000, there is an error with about 10 degrees in the estimated DOA. As the number of samples increases, the error in the estimated DOA reduces.

The performance with the conventional MUSIC and the proposed algorithm with 8 carriers is shown in Fig. 4. It is also clear that the conventional MUSIC algorithm does not work even though the number of carriers increases. As for the proposed algorithm, compared with Fig. 2, the error in the estimated DOA reduces due to the averaging of the correlation matrices over the different carriers. The DOA can be accurately identified even with 1000 samples/carrier. Therefore, with the proposed algorithm the number of signals whose direction is estimated can be increased and the accuracy of the DOA estimation can be improved by employing more number of carriers.

The DOA estimation performance of the conventional and the proposed algorithm on the Rician fading channel is shown in Figs. 5 and 6. $E_b/N_0$ is set to 10 [dB]. On the Rician fading channel, the performance of the proposed algorithm is deteriorated due to fading. Thus, the proposed algorithm with 8 carriers and even with 10000 samples/carrier estimates the DOA of the signals with large errors as shown in Fig. 5. The reason is that correlation matrix in Eq. (15) changes during the

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**Fig. 3** Direction finding through the conventional and the proposed algorithm, AWGN channel, $E_b/N_0 = 10$ [dB], 2 carriers, 10000 samples/carrier.

**Fig. 4** Direction finding through the conventional and the proposed algorithm, AWGN channel, $E_b/N_0 = 10$ [dB], 8 carriers, 10000 samples/carrier.

**Fig. 5** Direction finding through the conventional and the proposed algorithm, Rician fading channel, $E_b/N_0 = 10$ [dB], 8 carriers, 10000 samples/carrier.

**Fig. 6** Direction finding through the conventional and the proposed algorithm, Rician fading channel, $E_b/N_0 = 10$ [dB], 16 carriers, 1000 samples/carrier.
calculation of cumulants. Thus, taking more number of samples does not improve the accuracy of the DOA estimation. In this case, calculation of the correlation matrices with fewer samples and averaging them over the carriers are more effective. The DOA estimation performance with 16 carriers are shown in Fig. 6. In this figure, the DOA of the 3 signals can be estimated with smaller errors with 1000 samples/carrier.

5. Conclusions

In this paper, the novel cumulant based MUSIC like DOA estimation algorithm with multicarrier modulation has been proposed. The proposed algorithm can be applied to the received signals transmitted over the different carriers. The proposed algorithm does not require the sensor array responses for the frequency range of the interest and the initial phases of the carriers. Through the simulation results it has been shown that the proposed algorithm can estimate the DOA of the signals clearly on the AWGN channel. Though Rician fading deteriorates the DOA estimation performance of the proposed algorithm, the accuracy can be improved by employing larger number of carriers. With the proposed algorithm the number of signals whose directions are estimated can be also increased by utilizing larger number of carriers.

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References


Appendix: Derivation of Cumulants

Let \( \mu \) denote a collection of random variables, i.e., \( \mu = \{\mu_1, \mu_2, \ldots, \mu_q\} \), and \( I_p = 1, 2, \ldots, h \) denote the set of indices of the components of \( \mu \). If \( I \subset I_p \), then \( \mu_I \) is the vector consisting of those components of \( \mu \) whose indices belong to \( I \). We denote the simple moment and cumulant of the subvector \( \mu_I \) of the vector \( \mu \) as \( m_p(I) \) and \( C_p(I) \). The partition of the set \( I \) is the unordered collection of nonintersecting nonempty sets \( I_p \), such that \( \cup_{p=1}^q I_p = I \). For example, the set of partitions corresponding to \( h = 3 \) is \{\{(1,2,3)\}, \{(1)(2,3)\}, \{(2)(1,3)\}, \{(3)(1,2)\}, \{(1)(2)(3)\}\}. The moment-cumulant formula is:

\[
C_p(I) = \sum_{\cup_{p=1}^q I_p = I} (-1)^{q-1}(q - 1)!
\]

\[
\prod_{p=1}^q m_p(I_p)
\]

where \( \cup_{p=1}^q I_p = I \) denotes summation over all parti-
tions of set $I$.

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