Influence of timing jitter on quadrature charge sampling

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Abstract: The influence of timing jitter on quadrature charge sampling for a direct conversion receiver has been evaluated. In contrast to voltage sampling, the charge sampling mixer integrates the signal current instead of tracking the signal voltage. The charge sampling mixer has been applied to RF direct sampling in wireless receivers. The influence of timing jitter on charge sampling has been analysed in some literatures. However, in wireless receivers, quadrature sampling is required in order to demodulate I-phase and Q-phase signals. Different from charge sampling, timing jitter causes crosstalk between these signals. The influence of timing jitter on quadrature sampling is investigated.

1 Introduction

Recently, software defined radio (SDR), which enables one receiver to be used for various wireless systems, has been actively investigated. RF front-end and analog-to-digital converters (ADCs) are the key components of SDR systems. However, as there is no ADC that can be operated at RF, existing receivers cannot convert the received signal from the analog domain to the digital domain directly [1].

Lately, new receiver architectures have been proposed for possible application to SDR [2–6]. These receivers sample the received signal in RF. In these architectures, charge sampling mixers are used, and sampling and downconversion are carried out simultaneously. The baseband signal is then extracted through the discrete time analog filters.

The effect of timing jitter on charge sampling has been analysed, for example, in [7, 8]. It has been shown that the effect of timing signal to noise ratio (SNR) performance of charge sampling is different from that of voltage sampling. If the sampling jitter is small, the SNR of charge sampling is worse than that of voltage sampling. However, none of these literatures have assumed the application of the charge sampling mixer to wireless receivers. In the wireless receiver, the received signal consists of the I-phase and Q-phase components, which are sampled by quadrature sampling [4, 9]. The receiver may lose the orthogonality between the I-phase and Q-phase components because of timing jitter and suffer from crosstalk between them.

The influence of timing jitter on quadrature charge sampling for a direct conversion receiver is investigated. The signal to interference and noise ratio (SINR) of the downconverted signal is evaluated with different data rates and modulation schemes. In addition, the distributions of timing jitter based on the structure of the PLL have been modelled. In the literature, such as in [7, 8, 10], it is assumed that the output signal is synchronised before a new sample is taken. It is also shown that the SINRs of charge sampling and voltage sampling are almost equivalent when the jitter is sufficiently small.

2 System model

2.1 Receiver architecture

The receiver architecture assumed is shown in Fig. 1. In this architecture, the received signal is sampled with the quadrature charge sampler. The quadrature charge sampler samples the received signal at RF. The sampled are then input into the finite impulse response (FIR) filters. Here, it is assumed that all the coefficients of the FIR filters are set to 1.

2.2 Charge sampling circuit

The simple circuit of the charge sampler is shown in Fig. 2. The input voltage is first transformed into a corresponding current with a transconductance element. For simplicity, it is assumed that this element performs ideal V-to-I conversion. The output current of the transconductor is integrated into the sampling capacitor during a predetermined period with the sampling switch $P_{int}$. After the integration period, the output is sampled with the switch $P_{out}$, and the sampling capacitor is discharged with the switch $P_{res}$ before a new sample is taken.

2.3 PLL model

The model of the phase locked loop (PLL) system is shown in Fig. 3. It is assumed that the output signal is synchronised with that of the carrier signal for simplicity. This assumption holds for a direct conversion receiver because the amount of phase rotation of the demodulated signal can be detected in the digital domain, and phase synchronisation may be achieved ideally with a digitally controlled oscillator [6].
The output from the PLL contains the phase noise whose spectrum has the typical shape shown in Fig. 4. There are two major noise sources [11]. One is the voltage-controlled oscillator (VCO) and the other one is the temperature-compensated crystal oscillator (TCXO) [12]. The phase noise caused by the TCXO is low-pass-filtered by the transfer function of the PLL, and its power spectrum is concentrated at around the carrier frequency. On the other hand, the PSD of the phase noise caused by the VCO is much lower than that of the noise caused by the TCXO at around the output frequency of the PLL. It can be seen from Fig. 4 that the PSD value increases with frequency until it reaches a peak and then decreases in proportion to the logarithm of frequency. It is the dominant source of the phase noise if the frequency is far away from the output frequency.

For simplicity, it is assumed here that the PSD of the phase noise consists of two components as shown in Fig. 5 [11]. One has a white spectrum shape with a density of \( N_g \). This component is dominant at frequency regions lower and higher than the output frequency. The other one has a non-white spectrum shape with a density of \( N_n \) at the output frequency. This PSD can be modelled by a single-pole low-pass filter with a cutoff frequency of \( f_B \) [12]. This component is dominant at around the output frequency.

### 2.4 Influence of timing jitter

The influence of the phase noise of the PLL on the clock signal is described here. The output signal from the PLL is given as

\[
s_p(t) = \sin (\omega_c t) + n_p(t)
\]  

where \( n_p(t) \) is the PLL noise and \( \omega_c \) the angular frequency of the of the RF signal. This signal is input into the comparator, and the clock signal as shown in Fig. 6. Thus, the phase noise causes the clock jitter.

Assuming that \( \omega_c t = 2k\pi \) (where \( k \) is an integer)

\[
s_p(t) \approx \sin \left( \omega_c \frac{n_p(t)}{\omega_c} \right)
\]  

The clock jitter is then calculated as

\[
T(k) = \frac{n_p(2k\pi/\omega_c)}{\omega_c} = \frac{n_p(k/\omega_c)}{\omega_c} = n_p(k/\omega_c)
\]  

The clock jitter directly causes the sampling jitter.

### 3 Numerical analysis

Here the influence of the phase noise modelled in Section 2 on signal constellation with single-carrier quadrature phase shift keying (QAM) and orthogonal frequency division multiplexing (OFDM) modulation is analysed. The SNR and SINR are then derived.

#### 3.1 Single-carrier QAM

The transmitted signal is modulated with single-carrier QAM and transmitted over the additive white Gaussian
noise (AWGN) channel. The received signal is given as
\[
r(t) = s(t) + n(t)
\]
\[
= A_I(t)n_I \cos(\omega_c t) + A_Q(t)n_Q \sin(\omega_c t) + n(t)
\]
(4)
where \( A_I \) and \( A_Q \) are the amplitudes and \( n_I \) and \( n_Q \) are the information signals of the I-phase and Q-phase components, respectively.

The received RF signal is sampled as shown in Fig. 7. Here, the sampling process is modelled by the multiplication of the received signal with the rectangular signal and integration. It is also assumed that the phase of the rectangular signal is synchronised with the received carrier signal. Each sampled signal of the I-phase or Q-phase components is an integrated half-cycle of the carrier signal as shown in Fig. 8.

Here, the sampling jitter on the I-phase or Q-phase signals is represented as \( \tau(k) \). The narrow-band Gaussian noise \( n(t) \) is given by
\[
n(t) = n_I(t)\cos(\omega_c t) + n_Q(t)\sin(\omega_c t)
\]
(5)

When the symbol rate is \( f_s \), one symbol is transmitted from \(-1/4f_c\) to \([((f_s/f_c) - 1)/2] + (3/4)f_c\). The \( k \)-th sampled signal of the I-phase component \( r_I(k) \) is then given by
\[
r_I(k) = \frac{1}{\Delta} \int_{((k/f_c)-(\Delta/2))+\pi(k)}^{((k/f_c)+(\Delta/2))+\pi(k)} \{s(t) + n(t)\} \, dt
\]
\[
= (A_I(k)n_I + n_I(k)) \sin((f_s/f_c)\cos(\omega_c \tau(k)))
+ (A_Q(k)n_Q + n_Q(k)) \sin((f_s/f_c)\sin(\omega_c \tau(k)))
\]
(6)
where \( \Delta \) is the integration period, and \( A_I(k) \), \( A_Q(k) \), \( n_I(k) \) and \( n_Q(k) \) are given as
\[
A_I(k) = \int_{((k/f_c)-(\Delta/2))+\pi(k)}^{((k/f_c)+(\Delta/2))+\pi(k)} A_I(t) \, dt
\]
(7)
\[
A_Q(k) = \int_{((k/f_c)-(\Delta/2))+\pi(k)}^{((k/f_c)+(\Delta/2))+\pi(k)} A_Q(t) \, dt
\]
(8)
\[
n_I(k) = \int_{((k/f_c)-(\Delta/2))+\pi(k)}^{((k/f_c)+(\Delta/2))+\pi(k)} n_I(t) \, dt
\]
(9)
\[
n_Q(k) = \int_{((k/f_c)-(\Delta/2))+\pi(k)}^{((k/f_c)+(\Delta/2))+\pi(k)} n_Q(t) \, dt
\]
(10)
where \( \omega_c \tau(k) \) is much less than the cycle of the carrier signal. The following approximation can then be applied.
\[
\sin(\omega_c \tau(k)) \simeq \omega_c \tau(k)
\]
(11)
\[
\cos(\omega_c \tau(k)) \simeq 1 - \frac{\omega_c^2 \tau(k)^2}{2}
\]
(12)

These sampled signals of the I-phase and Q-phase components are input into the LPFs, respectively, the outputs of which are given by
\[
d_I = \sum_{k=0}^{(f_c/f_s)\tau-1} 2(\tilde{A}_I(k)m_I + n_I(k))
\]
\[
+ \frac{2(\tilde{A}_Q(k)m_Q + n_Q(k))}{\pi} \omega_c \tau(k)
\]
(13)
\[
d_Q = \sum_{k=0}^{(f_c/f_s)\tau-1} \frac{2(\tilde{A}_Q(k)m_Q + n_Q(k))}{\pi} (f_c/f_s)^2 \omega_c \tau(k)
\]
(14)
where
\[
\hat{\tau} = \frac{f_s}{f_c} \sum_{k=0}^{(f_c/f_s)\tau-1} \tau(k)
\]
(17)
\[ \tilde{A}_1 = \frac{2}{f_c} \sum_{k=0}^{(f_c/f_s)-1} A_i(k) \]  

(18)

\[ \tilde{A}_Q = \frac{2}{f_c} \sum_{k=0}^{(f_c/f_s)-1} A_q(k) \]  

(19)

\[ \tilde{n}_1 = \frac{2}{f_c} \sum_{k=0}^{(f_c/f_s)-1} n_i(k) \]  

(20)

\[ \tilde{n}_Q = \frac{2}{f_c} \sum_{k=0}^{(f_c/f_s)-1} n_q(k) \]  

(21)

Suppose that

\[ G = \frac{2(f_c/f_s)}{\pi} \]  

(22)

then (15) and (16) become

\[ d_i = G((\tilde{A}_1m_1 + \tilde{n}_1) + (\tilde{A}_Qm_Q + \tilde{n}_Q)(\omega_c \tilde{\tau})) \]  

(23)

\[ d_Q = G((\tilde{A}_Qm_Q + \tilde{n}_Q) - (\tilde{A}_1m_1 + \tilde{n}_1)(\omega_c \tilde{\tau})) \]  

(24)

where \( \tilde{n}_1 \) and \( \tilde{n}_Q \) are the white Gaussian noise with zero mean and a variance of \( N_0 \), respectively.

From (3), \( \tau \) is given as

\[ \tilde{\tau} = \frac{2}{f_c} \sum_{k=0}^{(f_c/f_s)-1} \tau(k) \]

\[ = \frac{2}{f_c} \sum_{k=0}^{(f_c/f_s)-1} \frac{n_i(k)}{\omega_c} \]

\[ = \frac{2}{f_c \omega_c} \sum_{k=0}^{(f_c/f_s)-1} n_p(k) \]

\[ = \frac{n_p}{\omega_c} \]  

(25)

Thus, (23) and (24) can be rewritten as

\[ d_i = G((\tilde{A}_1m_1 + \tilde{n}_1) + (\tilde{A}_Qm_Q + \tilde{n}_Q)(\omega_c \tilde{\tau})) \]  

(26)

\[ d_Q = G((\tilde{A}_Qm_Q + \tilde{n}_Q) - (\tilde{A}_1m_1 + \tilde{n}_1)(\omega_c \tilde{\tau})) \]  

(27)

From \( d_i \) and \( d_Q \), the QAM symbol is demodulated.

### 3.2 OFDM modulation

For the case of OFDM modulation, from (26) and (27), the \( i \)th sampled signal of the I-phase or Q-phase component is rewritten as

\[ d_i(i) = G((\tilde{A}_1m_1(i) + \tilde{n}_1(i)) + (\tilde{A}_Qm_Q(i) + \tilde{n}_Q(i))(\omega_c \tilde{\tau})) \]  

(28)

\[ d_Q(i) = G((\tilde{A}_Qm_Q(i) + \tilde{n}_Q(i)) - (\tilde{A}_1m_1(i) + \tilde{n}_1(i))(\omega_c \tilde{\tau})) \]  

(29)

\[ d(i) = [d_i(i) + jd_Q(i)] \]  

(30)

Therefore the demodulated signal on the \( l \)-th subcarrier can be written as

\[ z(l) = \frac{1}{N} \sum_{n=0}^{N-1} d(i) \exp \left( \frac{j2\pi li}{N} \right) \]  

(31)

### 3.3 SNR and SINR

As shown in (6) and (12), timing jitter reduces the received signal amplitude from \( A_i(k) \) to \( A_i(k)(1 - \omega^2 \tau(k)^2/2) \). From (17) and (18), the amplitude of the received I-phase signal is given as \( A_i(k)(1 - \omega^2 \tau(k)^2/2) \). Therefore the SNR of the sampled I-phase component is calculated as [7, 8]

\[ \text{SNR} = \frac{(\tilde{A}_1 + \tilde{n}_1)^2}{(\tilde{A}_1 + \tilde{n}_1)^2(\omega_c \tau^2/2) + ((\tilde{A}_Q + \tilde{n}_Q)(\omega_c \tau))^2} \]  

(32)

However, as shown in (6), for the case of quadrature sampling, there is a crosstalk component from the Q-phase signal. Therefore, the SINR of the received signal is calculated as

\[ \text{SINR} = \frac{(\tilde{A}_1 + \tilde{n}_1)^2}{((\tilde{A}_1 + \tilde{n}_1)^2(\omega_c \tau^2/2) + ((\tilde{A}_Q + \tilde{n}_Q)(\omega_c \tau))^2) (\tilde{A}_1 + \tilde{n}_1)^2(\omega_c \tau)^2)} \]  

(33)

### 3.4 Comparison of charge sampling and voltage sampling

From Fig. 8 and (6), the charge sampling of the I-phase component of the carrier signal with the integration period of \( \Delta \) and the timing jitter of \( \tau(k) \) is expressed as

\[ \text{SNR} = \frac{1}{\Delta} \int_{-\infty}^{\infty} \text{rect} \left( \frac{t - (k/f_c)}{\Delta} + \tau(k) \right) \cos(\omega_c t) \, dt \]

\[ = \frac{1}{\Delta} \int_{(k/f_c) - (\Delta/2)}^{(k/f_c) + (\Delta/2)} \cos(\omega_c t) \, dt \]  

(34)

where \( \text{rect}(at) \) is the rectangular pulse shape with a width of \( a \).

For charge sampling with the integration period of \( \Delta \), (34) is rewritten as

\[ \text{SNR} = \frac{1}{\Delta \omega_c} \left[ \sin(\omega_c t) \right]_{(k/f_c) - (\Delta/2) + \tau(k)}^{(k/f_c) + (\Delta/2) + \tau(k)} \]

\[ = \frac{1}{\Delta \omega_c} (\sin(2\pi k + \pi f_c \Delta + \omega_c \tau(k)) - \sin(2\pi k - \pi f_c \Delta + \omega_c \tau(k))) \]

\[ = \frac{2}{\Delta \omega_c} \sin(\pi f_c \Delta) \cos(\omega_c \tau(k)) \]

\[ = \sin(c(f_c \Delta)) \cos(\omega_c \tau(k)) \]  

(35)

The charge sampling of the Q-phase component is described as

\[ \int_{(k/f_c) - (\Delta/2) + \tau(k)}^{(k/f_c) + (\Delta/2) + \tau(k)} \sin(\omega_c t) \, dt \]

\[ = \frac{2}{\Delta \omega_c} \sin(\pi f_c \Delta) \sin(\omega_c \tau(k)) \]

\[ = \sin(c(f_c \Delta)) \sin(\omega_c \tau(k)) \]  

(36)

Then the SINR is given as in (33) with the condition of \( \Delta = 1/2f_c \).

On the other hand, as \( \Delta \) approaches 0, the rectangular pulse turns into Dirac’s delta function. Voltage sampling
is then expressed as
\[
\lim_{\Delta \to 0} \frac{1}{\Delta} \int_{-\infty}^{+\infty} \text{rect} \left( \frac{t - (k/f_c) + \pi(k)}{\Delta} \right) r(t) \, dt
\]
\[
= \int_{-\infty}^{+\infty} \delta \left( t - \left( k/f_c + \pi(k) \right) \right) r(t) \, dt
\]
\[
= r \left( k/f_c + \pi(k) \right) \tag{37}
\]
The SINR of the voltage sampling is then given as [8]
\[
\frac{E[r^2(k/f_c)]}{E[(r(k/f_c) + r(k/f_c))^2]}
\]
\[
\simeq \frac{E[r^2(k/f_c)]}{E[(r^2(k/f_c)/\tau^2(k))]}
\]
\[
= \frac{E[(A_1(k/f_c)m_1(k/f_c)]}{[E((-A_1(k/f_c)m_1(k/f_c)]}
\]
\[
+ n_1(k/f_c)^2 \cos^2(\omega_1(k/f_c))
\]
\[
+ n_2(k/f_c) \sin(\omega_2(k/f_c))
\]
\[
+ (A_0(k/f_c)m_0(k/f_c)) \cos^2(\omega_0(k/f_c)) \omega_0^2 \tau^2(k)
\]
\[
= \frac{(A_1 + \bar{A}_1)^2}{(A_0 + \bar{A}_0)^2 \omega_0^2 \tau^2} \tag{38}
\]
Comparing (33) and (38), depending on the phase of the carrier signal, it is clear that the SNR of voltage sampling can be much better than that of charge sampling with small timing jitter. However, in terms of the SINR, there is no significant difference between charge sampling and voltage sampling.

4 Numerical results

4.1 Simulation conditions

The influence of the modelled sampling jitter on the received signal is evaluated by computer simulation. The simulation conditions are shown in Table 1.

White noise components of the phase noise are assumed to have PSDs ranging from $150$ to $-100$ dBc/Hz. For example, the noise of the PLL proposed in [13] shows a PSD of $-110$ dBc/Hz. The non-white noise component is modelled by the single-pole low-pass filter with a cutoff frequency of 10 kHz. The PSD of this component, $N_n$, is simulated with $80$ dBc/Hz [13]. The simulation is conducted with the symbol rates ranging from 0.1 to 100 Msymbol/s. The influences of the sampling jitter on the SINR and BER performances are evaluated with single-carrier QAM and OFDM modulation.

4.2 Numerical results

4.2.1 SNR and SINR: Figs. 9 and 10 show the SNR and SINR of the I-phase component defined in (32) and (33) as functions of the symbol rate. $E_b/N_0$ is set to 14 dB. The SNR and SINR show the same performance curves for both modulation schemes. As the symbol rate decreases, both the SNR and SINR improve because of the noise reduction capability of the FIR filter. The SNR remains unchanged among three different modulation schemes, whereas the SINR largely depends on them. This is because of the fact that the SINR is defined by the crosstalk.
term between the I-phase and Q-phase components. With the same \( E_b/N_0 \), the crosstalk term becomes larger as the modulation index increases. Also, the SINR is significantly lower than the SNR. Thus, instead of the SNR, the SINR should be calculated for quadrature charge sampling in wireless receivers.

### 4.2.2 BER

The BER performance against \( E_b/N_0 \) with single-carrier 64QAM is shown in Fig. 11. The symbol rate is 100 Msymbol/s. In this figure, when \( N_g \) is more than \(-110 \text{ dBc/Hz}\), the BER performance is worse than the theoretical performance. This is because of the cross-talk component caused by the timing jitter in the quadrature charge sampling mixers.

This result can be confirmed with a simple approximation. Suppose that \( N_g \) is \(-100 \text{ dBc/Hz}\) and the symbol rate is 100 Msymbol/s. From (3), \( E[(\alpha_k \tau(k))] = -20 \text{ dB} \), and from (33), the SINR of the sampled signal is about 20 dB. When \( E_b/N_0 = 14 \text{ dB} \), that is, \( E_s/N_0 = 22 \text{ dB} \), the variance of the thermal noise is almost the same as that of the interference. Thus, the BER with the sampling jitter and \( E_b/N_0 = 14 \text{ dB} \) should be the same as the theoretical performance with \( E_b/N_0 = 11 \text{ dB} \), which is the result obtained in Fig. 11.

The BER performance against the symbol rate with single-carrier 64QAM is shown in Fig. 12. \( E_b/N_0 \) is set to 14 dB. As shown in this figure, as the symbol rate increases, the bit error rate (BER) increases. This is because the averaging effect of the FIR decreases as the bandwidth increases.

### 5 Conclusions

The influence of timing jitter on quadrature charge sampling has been derived and evaluated through computer simulation. Instead of the SNR, the SINR of the demodulated signal has been analyzed for wireless communication applications. It has been shown that timing jitter deteriorates the SNIR of the demodulated signal because of the crosstalk between the I-phase and Q-phase components of the received signal. The BER performances with both single-carrier QAM and OFDM modulation schemes have shown the error floor with higher data rates. Therefore in a wideband system, the timing jitter in quadrature charge sampling may limit the performance of the receiver. It has also been shown that the SINRs of charge sampling and voltage sampling are almost the same, whereas the SNR of voltage sampling is better than that of charge sampling.

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### 7 References