Multipath Diversity through Time Shifted Sampling for Spatially Correlated OFDM-Antenna Array Systems

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SUMMARY An essential condition for diversity reception is that the fading distributions between individual received signals of an antenna array are uncorrelated. In this paper, a new technique to improve the performance of transmission with the correlated Rayleigh-fading signals is proposed. In conventional array systems, individual receivers start sampling the received signals at the same time with the same sampling rate. On the other hand, in the proposed scheme, the received signals are again sampled at the same rate, however the sampling points are shifted in each receiver. Numerical results through computer simulation show that with correlated received signals, by applying the proposed technique the correlation can be reduced to a sufficient level for diversity reception.

key words: antenna array, fading correlation, OFDM

1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) has been adopted by many wireless standards such as digital video broadcasting, IEEE 802.11a. It is getting more popular because of its ability to convert a frequency selective multipath channel into a set of parallel frequency-flat fading channels. In case of a spatially multiplexed system, OFDM is found to be advantageous in terms of ergodic capacity in rich multipath environments [1]. For single antenna OFDM systems, it is also possible to extract multipath diversity through fractional sampling (FS) [2]. In a usual single antenna OFDM receiver, the sampling is carried out at the baud rate of \(1/T_s\) (\(T_s\) is the sampling interval). On the other hand, in FS, a received signal is sampled at the multiples of baud rate \((G/T_s)\), where \(G\) is some integer) and the resultant sampled signals are combined at the receiver. By this way, the system leads to SIMO and benefits from multipath diversity.

However, FS requires A/D converters with higher sampling rates which cause high power consumption [3]. It also suffers from intercarrier interference if the sampling rate is higher than the Nyquist rate. Also, when the number of multipaths is low, the achievable diversity order by FS is limited. Hence, in some real wireless scenarios, employing antenna arrays is more effective solution to overcome fading errors.

The capacity of antenna array systems is mainly limited by the spatial fading correlation between antenna elements [4]. This problem generally occurs on the uplink (between the antenna elements of a base station) since a mobile unit is usually surrounded by many scatterers which provide almost independent reception, while the base station is not. It has been shown that when the fades are independent at each element, antenna arrays provide an impressive gain to the capacity of transmission [5]. The results of [6] states that when maximum ratio combining (MRC) is employed, spatial correlation less than 0.5 can be considered uncorrelated. However, in real wireless channels, especially in poor scattering environments, it is difficult to keep spatial correlation level less than 0.5. This might require larger antenna separation which is not desired when the space is limited.

In this paper, we propose a new method of diversity reception with correlated fading distributions for OFDM antenna array systems. In conventional array systems, individual receivers start sampling the received signals at the same time with the same sampling rate. In the proposed scheme, the received signals are again sampled at the same rate, however, sampling points are shifted in each receiver. We call this scheme Time Shifted Sampling (TSS). Numerical results through computer simulation show that, with TSS, spatial correlation between antenna elements can be reduced by exploiting multipath diversity.

This paper is organized as follows. The method of generating correlated Rayleigh-fading signals for antenna arrays is presented in Sect. 2. Then, proposed TSS scheme is defined and investigated in Sect. 3. Numerical results through computer simulation are presented in Sect. 4. Finally, the paper is concluded in Sect. 5.

2. Spatially Correlated Rayleigh Channel Model

The received signal is modeled as multiplication of transmitted signal and a Rayleigh fading waveform. In case of antenna arrays, a set of fading waveforms is needed for each receiver to generate individual received signals.

It is known that fading correlation between the antenna elements strongly depends on the spatial aspects of the channel. A number of previous works propose different approaches to model the spatial channel. Using a scatterer model is usually a simple way to characterize the channel. A very detailed overview of the scattering models can be found in [7].

In this work, in order to obtain a set of correlated Rayleigh fading waveforms, the scatterer model based on Jakes’ ring model [8] is chosen.
In [9], a method of generating correlated fading waveforms for antenna arrays, by extending the Jakes ring model, is introduced. Figure 1 illustrates the model. This model is basically a ray-tracing model which is based on the physical characteristics of the channel, such as antenna spacing ($\delta$), antenna arrangement (linear, circular etc.), angle of arrival of cluster ($\Psi$) and angular spread of cluster ($2\Delta$). The amount of spread is determined by the radius of the ring ($R$) and the distance ($d$) between the receiver and the transmitter. The sum of each scattered signal is Rayleigh distributed. In case of multiple antennas, another correlated Rayleigh waveform can be generated by using the same scatterers. It is assumed that all rays arrive receiver array at the same time with equal power.

The scatterers are placed uniformly on the ring. Angle $\alpha$ for each scatterer is given as:

$$\alpha_s = \frac{2\pi (s - 0.5)}{S}$$

where $S$ is the number of scatterers on the ring and $s$ is the scatterer index. The Doppler frequency shift for each scattered signal is $w_s = \frac{2\pi}{c} f v \cos(\alpha_s - \zeta)$, where $v$ is the speed of mobile, $c$ is the speed of light, $f$ is the frequency, and $\zeta$ is the angle of motion of the mobile as shown in Fig. 1. Each scattered signal is summed to form a Rayleigh fading waveform. If one fading waveform experienced at one spot $x$ along the axis of array is $T(t, x)$;

$$T(t, x) = \frac{1}{\sqrt{S}} \sum_{s=1}^{S} \exp(j w_s t + \phi_s)$$

where $\phi_s$ is the random phase $(0, 2\pi]$, then the waveform at the adjacent antenna which is separated by $\delta$ (wavelength) becomes;

$$T(t, x + \delta) = \frac{1}{\sqrt{S}} \sum_{s=1}^{S} \exp(j w_s t + \phi_s) \cdot \exp(-j\delta 2\pi \sin(\phi_s)),$$

$\phi_s$ is the arriving angle of the $s$th scattered signal.

The correlation between $T(t, x)$ and $T(t, x + \delta)$ can be calculated as the time average product of the first waveform and the complex conjugate of the second waveform. Figure 2 shows the change in correlation coefficient with respect to antenna separation for different spatial channel characteristics.

As seen in Fig. 2, the fades between antennas become correlated as the antenna elements get closer to each other, the arriving angles of the scatterers get closer to the end-fire direction of the array or the angular spread of cluster decreases.

2.2 Multipath Ring Model for Antenna Arrays

Measurement results showed that incoming signals arrive generally within more than one cluster due to the far scattering environments [10]. In this work, a channel with two multipath components is used for simulating the performance of the proposed scheme in Sect. 4. The second Rayleigh component is generated by using another ring that circles the existing ring. Figure 3 illustrates this model. Following further assumptions are made in this model.

- All rays from both rings arrive the receiver array with equal power.
- The rays from the second ring arrive receiver array with an equal delay.

Fading waveforms are generated from both rings by...
the same procedure described in Sect. 2.1. Fading waveforms generated by the inner ring is said to be the first multipath component, and accordingly the outer is the second multipath component.

Simulation is run for two antennas in linear arrangement with antenna separation of 1 wavelength. The number of scatterers is chosen as 64 on both rings and the angle of arrival for both rings are set to 60 degrees. The angular spreads of the clusters from the inner and outer rings are taken as 5 and 10 degrees, respectively. (These parameters are referred to as Channel Model-1 in Sect. 4). The results have shown that the fading correlation between the antenna elements of the same multipath components is high. The spatial fading correlation coefficient between the first multipath components is 0.98 and between the second multipath components is 0.96, since angular spread is a bit larger for the second path. On the other hand, the fading correlation between the first and the second multipath components is still low. In Fig. 4, a sample impulse responses of the multipath channel for the both antennas and the fading correlation relationship are illustrated.

The fading envelopes generated by the simulation are given in Fig. 5. It can be seen that the fading correlation is high between the same multipath components at different antennas and it is low between the first and second multipath components.

3. Time Shifted Sampling

TSS is a signal processing technique that takes the benefit of multipath diversity, which already exists in the channel, in order to improve the system performance with spatially correlated received signals. Figure 6 gives the complete block diagram of the receiver that employs TSS.

Suppose OFDM symbol is transmitted and the information symbol on the kth subcarrier is \( s[k] \) \( (k = 0, 1, \ldots, N - 1) \). Then OFDM symbol is the IDFT of the information symbols,

\[
u[n] = \frac{1}{N} \sum_{k=0}^{N-1} s[k] e^{j2 \pi nk/N}
\]

where \( n = 0, 1, \ldots, N - 1 \) is the time index and \( N \) is the IDFT length. The baseband signal at the output of the filter is given by \( x(t) = \sum_{p=1}^{P-1} u[n] p(t - nT_s) \) where \( p(t) \) is the impulse response of the pulse shaping filter, \( T_s \) is the symbol duration and \( P \) is the sum of the IDFT length and the length of cyclic prefix. Then this signal is transmitted over different multi-path channels to different antennas \( i \) with impulse responses \( c_i(t) \) and is matched to the transmitted pulse shape. The received signals by each antenna \( i \) then become,

\[
y_i(t) = \sum_{n=0}^{P-1} u[n] h_i(t - nT_s) + v_i(t)
\]

where \( h_i(t) \) is the impulse response of the channel seen by antenna \( i \) and is given by \( h_i(t) = p(t) \star c_i(t) \star p(-t) \), \( \star \) denotes convolution. \( v_i(t) \) is the additive white Gaussian noise at the receivers. \( h_i(t) \) can be expressed in a baseband form as

\[
h_i(t) = \sum_{m=0}^{N_{mi} - 1} \gamma_{i,m} p_2(t - \tau_{i,m}).
\]

\( p_2(t) = \int p(t') p(t + t') dt' \) is the cross correlation of \( p(t) \), \( N_{mi} \) is the number of multipath components for antenna \( i \), \( \gamma_{i,m} \) is the complex gains of the \( m \)th component at the \( i \)th antenna and \( \tau_{i,m} \) is the path delay. Amplitudes of the channels are assumed time-invariant during one OFDM symbol.

When sampling is carried out at the baud rate of \( 1/T_s \) with a delay of \( (i - 1)T_s/M \) at antenna \( i \) over total of \( M \) antennas, the received signals in discrete-time are expressed as,

\[
y_i[n] = \sum_{l=0}^{P-1} u[l] h_i[n - l] + v_i[n]
\]

where \( y_i[n] = y_i(nT_s + (i - 1)T_s/M), h_i[n] = h_i(nT_s + (i - 1)T_s/M), v_i[n] = v_i(nT_s + (i - 1)T_s/M) \). At each antenna, noise samples are still independent and white. However the fading correlation between any antenna elements \( i \) and \( j \), \( h_i[n] \) and \( h_j[n] \), and accordingly between \( y_i[n] \) and \( y_j[n] \), is reduced. Channel gains in discrete-time at each antenna are given by,
Ts fadings correlation is low between the different multipath components. Hence, the correlation between the channel gains in the frequency domain is lowered for the array with TSS.

The basic input-output relationship in OFDM-antenna array systems is;

$$z[k] = H[k]s[k] + w[k]$$  \hspace{1cm} (10)

where $z[k]$, $H[k]$ and $w[k]$ are the Mx1 column vectors of the received symbols, the channel gains and the Gaussian noises at each subcarrier, respectively. Then the estimate of $s[k]$ through MRC combining is;

$$\hat{s}[k] = \frac{\hat{H}^H[k]z[k]}{\hat{H}^H[k]\hat{H}[k]}$$  \hspace{1cm} (11)

where $\hat{H}[k]$ is the estimated Mx1 channel column vector. Since MRC is employed in frequency domain, the amount of spatial correlation is also calculated in frequency domain. Spatial correlation between any antenna elements $i, j$ can be calculated as,

$$\rho[k]_{i,j} = \frac{[R_H[k]]_{i,j}}{\sqrt{E[|H[k]|^2]E[|H[j][k]|^2]}}$$  \hspace{1cm} (12)

The covariance function $[R_H[k]]_{i,j} = E[H[k]H^*[j][k]]$ can be written as;

$$[R_H[k]]_{i,j} = E\left\{ \sum_{n_1=0}^{N-1} \sum_{n_2=0}^{N-1} e^{-j2\pi(n_1-n_2)k/N} \prod_{m=0}^{Nm-1} \gamma_{i,m} \gamma_{j,m} \right\}$$

$$\times \sum_{m_1=0}^{Nm_1-1} \sum_{m_2=0}^{Nm_2-1} (\gamma_{i,m_1})^2 (\gamma_{j,m_2})^2 p_2(2n_1T_s + (i-1)T_s/M - \tau_{i,m_1})$$

$$\times p_2(2n_2T_s + (j-1)T_s/M - \tau_{j,m_2})$$  \hspace{1cm} (13)

As seen in Eqs. (12) and (13), when TSS is applied, spatial correlation between the antennas depends on the carrier index $k$ and the pulse shape $p_2(t)$. $\rho[k]_{i,j}$ can be obtained by
Monte Carlo simulations or by numerical computations.

If the example presented in Fig. 7 is revisited, the angle of arrival in the frequency domain are: \( H_1[k] = \gamma_{1,0} + \gamma_{1,1}p_2(-T_s/2) + e^{-j2\pi k/N}\gamma_{1,1}p_2(T_s/2) \) and \( H_2[k] = e^{i2\pi k/N}\gamma_{2,0}p_2(-T_s/2) + \gamma_{2,0}p_2(T_s/2) + \gamma_{2,1} \). The covariance function \([R_H[k]]_{1,2}\) then becomes:

\[
[R_H[k]]_{1,2} = E[e^{-j2\pi k/N}\gamma_{1,0}p_{2,0}2(-T_s/2) + \gamma_{1,1,0}p_{2,0}2(T_s/2) + e^{-j2\pi k/N}\gamma_{1,1}\gamma_{2,0}p_2(-T_s/2)^2 + \gamma_{1,1,1}\gamma_{2,0}2p_2(-T_s/2)p_2(T_s/2) + e^{-j2\pi k/N}\gamma_{1,1}\gamma_{2,1}p_2(T_s/2)^2 + e^{-j2\pi k/N}\gamma_{1,1}\gamma_{2,1}2p_2(T_s/2)] \tag{14}
\]

4. Numerical Results

Performance of the proposed TSS technique is evaluated through computer simulation. In each simulation, maximal ratio combining is employed and perfect channel estimation is performed. The pulse shaping filter \(p_2(t)\) is chosen as truncated sinc pulse of length \( 2T_s \). Table 1 shows the other parameters that are used in each simulation.

Simulations are run for two different spatial channel models. Model-1 stands for a very high spatial correlation scenario (\( \sim 0.98 \)) and Model-2 for more reasonable amount of correlation scenario (\( \sim 0.85 \)). In Model-1, antennas are displaced by 1.4 (wavelength). Whereas in Model-2, the displacement is increased to 1.2 which helps spatial correlation to reduce in Model-2. Measurement results in [11] show that when the base station is placed over the rooftop level, angular spread is usually less than 10° when it is placed below the rooftop level, angular spread can be measured as high as 20°. Therefore, in Model-1, angular spreads are chosen less than 10° to increase the spatial correlation between array elements. On the other hand, angular spreads are considered as wider for Model-2. In practise, the angle of arrival depends on the mobile unit position. As seen in Fig. 2, the spatial fading correlation is low when the angle of arrival is close to the front direction of the antenna array. Consequently, in Model-1 for very high spatial fading correlation, angle of arrival is chosen as 60° from the front direction and in Model-2 to lower the amount of correlation, angle of arrival is chosen as the front direction. Table 2 shows these two physical channel models that are used in simulations.

In Fig. 8, under Channel Model-1, the amount of spatial fading correlations between antenna elements for each subcarrier is presented. When TSS is employed, correlation can be reduced to a sufficient level for uncorrelated reception. As seen in Fig. 8, the fading correlation is lowered mostly around the middle subcarrier. The channel delay profiles in the simulations are very similar to the ones presented in Fig. 7. For that channel profile, the covariance of the frequency responses of the channels experienced by each antenna is given by Eq. (14). When \( k = N/2 \), also remembering that the \( e^{-j\pi} = -1 \) and \( p_2(t) = p_2(-t) \), all the terms except \( \gamma_{1,0}\gamma_{2,1} \) will cancel each other. Therefore, the amount of spatial correlation on the subcarrier index \( N/2 \) equals to the correlation between the first multipath component of the first antenna (\( \gamma_{1,0} \)) and the second multipath component of the second antenna (\( \gamma_{2,1} \)) which is close to the zero as explained before. Hence, under this channel condition, the fading correlation is expected to be the lowest around the middle subcarrier.

Figure 9 shows the improvement in BER performance with TSS for both spatial channel models in Table 2. Performance improvement for the system under Channel Model-1 is observed higher than the one under Channel Model-2. This is because there is no linear relationship between cor-

### Table 1 Simulation conditions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Antennas</td>
<td>2</td>
</tr>
<tr>
<td>Modulation Scheme</td>
<td>OFDM/QPSK</td>
</tr>
<tr>
<td>FFT size</td>
<td>64</td>
</tr>
<tr>
<td>Number of Subcarriers</td>
<td>64</td>
</tr>
<tr>
<td>Channel Model</td>
<td>2-path Rayleigh (Equal power)</td>
</tr>
<tr>
<td>Delay of Second Path</td>
<td>( T_s/2 )</td>
</tr>
<tr>
<td>OFDM Symbol Duration</td>
<td>64( T_s )</td>
</tr>
<tr>
<td>Guard Interval</td>
<td>16( T_s )</td>
</tr>
<tr>
<td>Doppler Frequency ( f_d T_s )</td>
<td>( 10^{-4} )</td>
</tr>
<tr>
<td>Channel Estimation</td>
<td>ideal</td>
</tr>
<tr>
<td>Combining Scheme</td>
<td>MRC</td>
</tr>
</tbody>
</table>

### Table 2 Spatial channel models used in simulations.

<table>
<thead>
<tr>
<th></th>
<th>Model-1</th>
<th>Model-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antenna Separation ( \delta )</td>
<td>1.1</td>
<td>1.2</td>
</tr>
<tr>
<td>Angular Spread of First Path ( \Delta_1 )</td>
<td>5°</td>
<td>10°</td>
</tr>
<tr>
<td>Angular Spread of Second Path ( \Delta_2 )</td>
<td>10°</td>
<td>15°</td>
</tr>
<tr>
<td>Angle of Arrival of Paths ( \Phi )</td>
<td>60°</td>
<td>0°</td>
</tr>
</tbody>
</table>
Fig. 9 BER performance improvement with TSS for two elements antenna array.

Fig. 10 BER performance improvement with TSS for Channel Model-2 vs. antenna separation. Eb/No is 20 dB.

Fig. 11 BER performance improvement with TSS for Channel Model-2 vs. angular spread. Eb/No is 20 dB.

Fig. 12 BER performance improvement with TSS for Channel Model-1 vs. angle of arrival. Eb/No is 20 dB.

relation coefficient and system performance. For example, system performance difference between correlation values of 0.99 and 0.9 is much higher than the difference between 0.89 and 0.8. BER performances for some different correlation coefficients can also be found in [12].

In Fig. 10, Channel Model-2 is considered and antenna separation is changed from 0 to 9λ (wavelength). The system which employs TSS performs better than the conventional system until the separation of 4λ. For larger separations, the performances are almost the same. After 4λ, the spatial fading correlation between the antennas is already low enough for diversity reception and employing TSS can not improve the performance further.

In Fig. 11, the angular spread of clusters are changed from 0° to 90° for both clusters. When the rays in clusters arrive within less than 20°, there is performance improvement observed with TSS. After 20°, the correlation is already low and the system achieves diversity so TSS can not add more improvement after that point.

In Fig. 12, the angle of arrivals of both clusters are changed from 0° (broad-side direction) to 90° (end-fire direction). At any value of the angle of arrival, there is improvement with TSS. When the arriving angle is 90°, system performance with TSS is the same as it is 0° for the conventional arrays.

The effect of the Doppler frequency on the system performance can be found in Fig. 13. Simulations are performed under the time variant channels during one OFDM symbol for faster fading channels with normalized Doppler frequencies $f_{DT}$ from $10^{-4}$ to $10^{-1}$. The system with TSS outperforms the conventional system under the both spatial channel models for the faster varying fading channels.

In Fig. 14, simulation is run for the same conditions in Table 1 and Table 2, however this time number of elements in array is taken as four. Again, the system with TSS can take more advantage of diversity reception and improve system performance.
Fig. 13  Effect of the normalized Doppler frequency on the BER performance for $E_b/No$ is 20 dB.

Fig. 14  BER performance improvement with TSS for four elements antenna array.

5. Conclusions

The spatial fading correlation is the main limitation factor in multiple antenna reception. For some cases, although the antenna elements are separated several times the wavelength, fading correlation is still high. Simulation results showed that TSS is effective technique to overcome the degradations in performance due to the spatial correlation. By Time Shifted Sampling, it is possible to place antennas closer to each other for the same performance outcome or system performs better with the existing antenna placement. The proposed technique can be applied to arrays with any number of elements and MIMO systems as well.

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References


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