Delayed Correlation Based Signal Detection Scheme with Filter Bank for OFDM Signal

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SUMMARY  Delayed correlation has been used to detect orthogonal frequency division multiplexing symbols with cyclic prefix in spectrum sensing. Because of the frequency offset, the outputs of the delayed correlation do not lie only on the real axis of a complex plane. Therefore, the absolute value of the outputs of the delayed correlation is employed. Furthermore, with the use of a filter bank, the number of the outputs of the delayed correlators increases and the averaging over the outputs decreases the noise variance. This paper proposes a new delayed correlation scheme that uses a filter bank and employs the absolute of the outputs of delayed correlation. The proposed scheme improves the probability of detection as the number of the branches of the delayed correlators increases. In the case of 6 branches, the proposed scheme reduces the required sample energy by 1 dB the probability of detection of 0.9.

key words: spectrum sensing, delayed correlation

1. Introduction

With the global spread of the Internet and the evolution of digital signal processing technologies in recent years, the demands for high-speed and large-capacity wireless communications have been increasing [1]. At the same time, the popularity of wireless communications has caused frequency spectrum shortages. Therefore, cognitive radio (CR) has been attracting much attention among researchers of wireless communications [2]–[4]. CR is a technology with which a mobile terminal recognizes the status of backbone networks and radio environments autonomously. CR promotes the effective use of a frequency resource by finding a channel spectrum that is temporarily not in use by other wireless systems [5].

In the CR technology, signal detection is important since the prevention of interference to the other users is the first principle. Some signal detection schemes use the cyclic prefix (CP) of an orthogonal frequency division multiplexing (OFDM) symbol [5]. In [6] the second order cyclostationarity of OFDM symbol due to the CP is used for signal detection. This scheme incurs large computation complexity. As a low complexity scheme, a delayed correlation based OFDM signal detection scheme has been investigated [7], [8]. As far as the authors know, none of these studies have included the frequency offset effect [9]. Carrier frequency offset rotates the phase of the delayed correlation outputs. If carrier frequency offset exists, the absolute values of the detection outputs have to be taken to compare against the threshold for signal detection.

In this paper, a delayed correlation based signal detection scheme with a filter bank is proposed. The summation of the absolute values of the delayed correlation following analysis filters reduces the variance of the noise auto-correlation outputs. Therefore, the threshold for the same probability of the false alarm can be decreased and the probability of OFDM signal detection improves.

This paper is organized as follows. The system model of the conventional and proposed signal detection schemes are explained in Sect. 2. The probability of detection using the conventional and proposed schemes are evaluated through computer simulation in Sect. 3. Finally, our conclusions are presented in Sect. 4.

2. System Model

2.1 System Model

The assumed system model is shown in Fig. 1. Multiple IEEE 802.11g/n terminals communicate each other in the Industry-Science-Medical (ISM) band and the transmit signals are modulated with OFDM. If the channel is crowded with the multiple terminals and the access points (APs), the transmit signals interfere one another. The AP then conducts the measurement of radio environments using a signal detection scheme on all the ISM channels. Based on the measurement, the AP finds a vacant channel and accommodates terminals. It is assumed that the AP knows the length of the OFDM symbol and the center frequency of the channel. However, there may be the frequency offset between
the carrier signal of the terminal and the local signal in the receiver of the AP.

2.2 Received Signal Model

In the transmitter, the sampling rate is set to twice as fast as the Nyquist rate of the signal. The transmit OFDM signal is then given as follows;

\[ x[n] = \sum_{k=-N}^{N-1} X[k] \exp \left( j2\pi nk \frac{N}{2N} \right) \]  

(1)

where \( n \) is the time index, \( k \) is the subcarrier index, \( N \) is the number of subcarriers within a channel. The transmit symbol, \( X[k] \), does not exist at the outside of the channel as follows;

\[ X[k] = 0 \begin{cases} -N < k < -\frac{N}{2} \\ \frac{N}{2} \leq k < N - 1 \end{cases} \]  

(2)

The last part of OFDM symbol is copied and appended at the beginning as a cyclic prefix (CP) and it is described as

\[ x[-N_C + i] = x[i + 2N - 1 - N_C], \quad i = 0, \ldots, N_C - 1 \]  

(3)

where \( 2N \) is OFDM symbol length and \( N_C \) is the CP length. At the receiver side, there may be a frequency offset in the received signal and it is given as

\[ y[n] = \left( \sum_{i} h[i] x[n-i] \right) \exp \left( j2\pi n \frac{N}{2N} \right) + v[n] \]  

(4)

where \( h[i] \) is the \( i \)-th impulse response of the channel including the transmit filter, \( v[n] \) is the white noise, and \( \alpha \) is the frequency offset normalized by subcarrier bandwidth.

2.3 Conventional Signal Detection with Delayed Correlation

A block diagram of the conventional signal detection scheme is shown in Fig. 2. The conventional signal detection scheme, OFDM signal is detected using the CP through delayed correlation. The received signal is downconverted to a baseband and filtered by a low pass filter. A raised cosine filter is used as the low pass filter and the \( i \)-th impulse response of the filter, \( h[i] \), is given as follows;

\[ h[i] = \frac{1}{2} \sin \pi i / 2 \cos [\pi \beta i / 2] / \pi i / 2 \left[ 1 - 4\beta^2(i/2)^2 \right] \]  

(5)

where \( \beta \) is the roll-off factor. The output of the delayed correlation, \( c[n] \), is given as follows;

\[ c[n] = \sum_{q=0}^{N_C-1} y[n-q] y[n-2N-q] \]  

\[ = \sum_{q=0}^{N_C-1} \left\{ \sum_{i} h[i] h[i] x[n-q-i] \right\} \exp \left( j2\pi n \frac{N}{2N} \right) + \sum_{i} h[i] h[i] x[n-q-i] \]  

\[ \exp \left( j2\pi n \frac{N}{2N} \right) + \sum_{i} h[i] h[i] \exp \left( j2\pi n \frac{N}{2N} \right) \]  

(6)

where * represents complex conjugate, \( q \) is the index for the summation over the CP length, and \( \gamma[n] \) represents the \( n \)-th received signal at the output of the low pass filter as

\[ \gamma[n] = \sum_{i} h[i] y[n-i] \]  

(7)

Because of the frequency offset as well as the delay paths on the channel including the impulse responses of the filters, the outputs of the delayed correlation do not lie only on a real axis of a complex plane. Therefore, the absolute value of the delayed correlation, \( |c[n]| \), is compared with the threshold \( \gamma \).

\[ |c[n]| \geq \gamma \]  

(8)

When the CP of the OFDM signal reaches the multiplier through the delay line in the delayed correlator, the relationship \( x[n-q-i-i] = x[n-2N-q-i-i] \) for \( N-N_C \leq n-2N-q-i-i \leq N-1 \) holds when \((i+i) = (i' + i')\). The terms that satisfy the condition of \((i+i) = (i' + i')\) in Eq. (6) turns to

\[ \sum_{q=0}^{N_C-1} \sum_{i} \sum_{i'} \sum_{i''} h[i] h[i] h[i'] h[i'] \cdot x[n-q-i-i] \exp \left( j2\pi n \frac{2N}{2N} \right) \]  

\[ \exp \left( j2\pi n \frac{2N}{2N} \right) \]  

(9)

Especially when \( i = i' \) and \( i = i' \), the part of Eq. (9) becomes

\[ \sum_{q=0}^{N_C-1} \sum_{i} \left| h[i] \right|^2 \left| h[i] \right|^2 \]  

\[ \cdot x[n-q-i-i] \exp \left( j2\pi n \right) \]  

(10)

and these terms lie on the same direction as the vector
exp\( (j2\pi \alpha) \). As shown in Eq. (10) the frequency offset effects on the outputs of the delayed correlation.

2.4 Proposed Signal Detection with Delayed Correlation

In the proposed signal detection scheme, the received signal is divided in a frequency domain by a filter bank and the delayed correlation is carried out on each branch. The outputs of the delayed correlators are added together and compared with the threshold. A block diagram of the proposed signal detection scheme is shown in Fig. 3. As the analyzer filter of the filter bank, which divides the frequency components of the received signal, a raised cosine filter is used. The sub-band frequency allocation of the filter bank is shown in Fig. 4. The number of branches is \( M \) and the impulse response of the \( m \)th analyzer filter, \( h_{rc}^m[i] \), is given as follows;

\[
h_{rc}^m[i] = \frac{1}{2M} \sin \pi i/2M \cos[\pi \beta i/2M]\frac{1}{1 - 4\beta^2[i/2M]^2} \cdot \exp\left( j\pi \frac{2m - (M + 1)}{2M} i \right).
\]

Consequently, the output of the \( m \)th delayed correlator is calculated as follows;

\[
c^m[n] = \sum_{q=0}^{Nc-1} \sum_{i} h_{rc}^m[i] h_{ic}^m[n - 2N - q]
\]

\[
= \sum_{q=0}^{Nc-1} \left( \sum_{i} \sum_{i'} h_{rc}^m[i'] h_{ic}^m[n - 2N - q - i'] \cdot \exp\left( j2\pi \alpha \frac{n - 2N - q - i'}{2N} \right) + \sum_{i} h_{rc}^m[i] v[n - 2N - q - i] \right)
\]

\[
\sum_{i} \sum_{i'} h_{rc}^m[i'] h_{ic}^m[n - 2N - q - i'] \cdot \exp\left( j2\pi \alpha \frac{n - 2N - q - i'}{2N} \right)
\]

\[
+ \sum_{i} h_{rc}^m[i] v[n - 2N - q - i].
\]

\[
\gamma^M[n] \text{ represents the } M \text{th received signal at the output of the } M \text{th sub-band of the filter bank and it is given as}
\]

\[
\gamma^M[n] = \sum_{i} h_{rc}^m[i] v[n - 2N - q - i].
\]

In the proposed signal detection scheme, the absolute value of the outputs of the delayed correlators are summed and compared with the threshold \( \gamma^M \).

\[
c^M[n] = \sum_{m=1}^{M} |c^m[n]| \geq \gamma^M.
\]

When the CP of the OFDM symbol reaches the multiplier through the delay line on each branch, the relationship \( x[n - q - i - i'] = x[n - 2N - q - i - i] \) holds. A peak output appears in \( |k^M[n]| \) because of the following terms with the same conditions as Eq. (10).

\[
\sum_{m=1}^{M} \sum_{q=0}^{Nc-1} \sum_{i} |h_{rc}^m[i]|^2 |h_{ic}^m[n - 2N - q - i]|^2 \exp(j2\pi \alpha).
\]

If no OFDM signal is included in the received signal, no peak output appears. If the OFDM signal is received, the same as Eq. (10), these terms lie on the direction of \( \exp(j2\pi \alpha) \).

2.5 Decision Threshold

The threshold is determined based on the variance of the noise when no signal is received. Without OFDM signal the sum of the outputs of the \( M \) delayed correlators are give as follows.

\[
c^M[n] = \sum_{m=1}^{M} \sum_{q=0}^{Nc-1} \sum_{i} \sum_{i'} h_{rc}^m[i'] v[n - 2N - q - i'] \cdot |h_{rc}^m[i]|^2 \exp(j2\pi \alpha).
\]

For \( M = 1 \), Eq. (16) also corresponds to the conventional scheme. Among the outputs obtained during the detection period, the maximum value is selected. This process is repeated \( N_F \) times. With \( N_F \) outputs of Eq. (16), the \( P_{FA} \cdot N_F \) th from the maximum is used as the threshold where \( P_{FA} \) is the probability of false alarm. \( P_{FA} \) is set to 0.1 [10].

The measured probability of false alarm for the different numbers of branches, \( M \), is shown in Fig. 5. It is assumed that only the additive white Gaussian noise (AWGN) exists on the channel. The signal length, \( N_F \), is set to 960 samples, \( N_T \) is set to 100000, and the filter length is \( 128T_s \), where \( T_s \) is the sampling interval.
The normalized histograms of the maximum of $N_F$ outputs for the conventional scheme ($M=1$) and the proposed scheme ($M=6$) are in Figs. 6 and 7. It is assumed that the number of branches $M=1$ or $M=6$, $N_F$ is 960 samples, $N_T$ is 100000, the filter length is $128\tau_s$, and the outputs are normalized by their average. When only the noise is included, the average power of the maximum outputs in the proposed scheme increases more than that of the conventional scheme as presented in Figs. 6(a) and 7(a). On the other hand, when the signal and the noise exist, the average power of the maximum outputs in the proposed scheme decreases more than that of the conventional scheme. However, from those figures, it is also clear that the maximum outputs are more concentrated at around the average in the proposed scheme and it reduces the overlapped area between the histograms of the noise and the signal-plus-noise as shown in Figs. 6(b) and 7(b). Thus, the threshold value can be reduced and the proposed signal detection with the filter bank improves the detection probability.

The relationship between the normalized variance of the noise and the number of branches is presented in Fig. 8. The variance of $\tilde{e}_M[n]$ decreases as the number of branches, $M$, increases as shown in Fig. 8. On the other hand, as increasing the number of branches, the circuit scale also increases. To clarify the limit of the proposed scheme the number of branches, $M$, has been increased up to 80 as shown in Fig. 8. As a result, the normalized variance converges to about 0.25 for $M \geq 40$.

The relationship between the variance of the noise and the filter length is shown in Fig. 9. The variance of the noise is normalized by that with the filter length of $128\tau_s$ in the conventional scheme. Figure 9 also indicates that the normalized noise variance converges for the filter length of
more than 128\(T_s\). It is a trade-off between the complexity and the amount of the reduction in the noise variance. In the following section the number of branches is set to up to 6 and the filter length is fixed to 128\(T_s\). This corresponds to the complexity of 2 DFT processes in terms of the number of multiplications in OFDM receiver.

3. Numerical Results

3.1 Simulation Conditions

Table 1 shows the simulation conditions. The transmit signal follows the format of the IEEE 802.11g standard. The information bits are modulated with QPSK and multiplexed with OFDM. The number of the subcarriers and the number of the data subcarriers are 64 and 52. The OFDM symbol duration is 3.2 \(\mu s\) and the CP length is 0.8 \(\mu s\). The tap coefficients of the transmit filter are calculated from the Fourier transform of the 11g spectrum mask with a Hamming window. The filter length is 150\(T_s\) and the bandwidth is 20 MHz where the sampling rate is 1/\(T_s\) = 40 MHz. The receive filter is the raised cosine filter. As channel models, an additive white Gaussian noise (AWGN) channel, an Indoor Residential-A channel, and an Indoor Office-B channel are assumed. The detection period for signal sensing is set to 6 \(\mu s\), 24 \(\mu s\), or 46 \(\mu s\). \(P_{FA}\) is set to 0.1 [10]. The number of trials for each plot is 10\(^5\).

3.2 Effect of Frequency Offset on Correlation Outputs

Figures 10 and 11 show the outputs of the delayed correlators in the conventional and proposed schemes. The received signal consists of continuous OFDM symbols without AWGN and no fading is assumed in these figures. The number of detection outputs are 1500 in all the figures. The outputs are normalized by the maximum amplitude among them in each figure. When no frequency offset exists, the correlation outputs spread around the real axis in Figs. 10(a) and 11(a). On the other hand, the correlation outputs spread around the imaginary axis due to the normalized frequency offset of \(\alpha = -0.25\) in Figs. 10(b) and 11(b). In Eq. (10) and Eq. (15), the output of the delayed correlation is a complex value owing to the frequency offset and the outputs do not always spread around the real axis. From these figures, it is clear that the absolute values of the delayed correlation should be taken and compared to the threshold for signal detection. It is also shown in Figs. 10 and 11 that the outputs for the proposed scheme are more correlated between the consecutive outputs than those for the conventional scheme.

3.3 Frequency Offset versus Probability of Detection

The effects of frequency offset on the probability of detection are shown in Figs. 12–14. The AWGN, the Indoor Residential-A, and the Indoor Office-B are assumed as channel models. \(E_s/N_0\) is set to \(-8\) dB, \(-5\) dB, and \(-3\) dB, respectively, where \(E_s\) is the energy of each sample and \(N_0\) is the noise spectrum density. “Real” means that the real part of the outputs of the delayed correlation is processed and “Abs” implies that the absolute values of the delayed correlation are processed. In Figs. 12–14, the probability of detection is calculated for each frequency offset and the threshold for signal detection is 0.1. The results show that the proposed scheme has a lower probability of detection than the conventional scheme for all channel models. The probability of detection decreases as the frequency offset increases, which is consistent with the simulation results.
correlation are taken and compared with the threshold. The number of samples in the detection period is 960. If only the real part of the correlation outputs are used, the conventional scheme shows the better detection probability than those with the proposed scheme. As the normalized frequency offset increases the probabilities of detection both with the conventional and proposed schemes deteriorate. On the other hand, the proposed scheme outperforms the conventional scheme if the absolute values are used for detection. Also the detection performance does not change according to the normalized frequency offset. On the Indoor Office-B channel, the probability of detection reduces both with the conventional and proposed schemes. This is because a large number of multipath components induce more cross terms with the conditions of $i' \neq i$ and $i'' \neq i_c$ in Eqs. (6) and (12) and those cross terms spread the correlation outputs over the complex plane in Figs. 10 and 11.

3.4 Number of Branches versus Probability of Detection

The relationship between the numbers of the delayed correlation branches and the probability of detection on different channel models are shown in Fig. 15–17. The normalized frequency offset is $-0.25$ and the number of samples in the detection period is 960. It is clear from the figures that the probability of detection improves as the number of the branches increases. This is because the averaging over the outputs of the branches reduces the noise variance. About 0.05 improvement can be observed with the proposed scheme of 6 branches at around the probability of detection of 0.9.

**Fig. 10** Correlation outputs of conventional detector ($M = 1$, Number of samples: 1500).

**Fig. 11** Correlation outputs of proposed detector ($M = 6$, Number of samples: 1500).

**Fig. 12** Probability of detection vs. frequency offset (AWGN channel, $E_s/N_0 = -8$ dB, $N_0 = 960$).
3.5 Required $E_s/N_0$ versus Detection Period

The relationships between the required $E_s/N_0$ for the probability of detection of 0.9 versus the detection period on different channel models are shown in Figs. 18–20. The detection periods are set to $6\mu$sec ($N_F = 240$), $24\mu$sec
Fig. 19 Relationship between observation period and $E_s/N_0$ (Indoor Residential-A channel, $\alpha = -0.25$).

Fig. 20 Relationship between observation period and $E_s/N_0$ (Indoor Office-B channel, $\alpha = -0.25$).

$(N_F = 960)$, or $46 \mu s (N_F = 1840)$. It can be observed that the probability of detection improves as the detection period increases. The required $E_s/N_0$ for the proposed scheme with the number of branches of 6 is smaller than that of the conventional scheme and the difference is larger with the smaller detection period.

4. Conclusions

In this paper, a signal detection scheme that uses delayed correlation with the filter bank has been proposed and applied to OFDM signals with frequency offset. In the proposed signal detection scheme, the received signal is divided in the frequency domain by the filter bank. With the use of the filter bank, the number of the outputs of the delayed correlators increases and the averaging over the outputs decreases the noise variance. Numerical results obtained through computer simulation have shown that the proposed scheme achieves the better probability of detection than that of the conventional scheme even when the carrier frequency offset exists. It has been clear that the probability of detection improves as the number of the branches of the delayed correlators increases. In the case of 6 branches, the proposed scheme reduces the required $E_s/N_0$ by 1dB for the probability of detection of 0.9 as compared with the conventional scheme when the detection period is shorter than 46$\mu s$. The amount of improvement in the required $E_s/N_0$ increases as the detection period decreases.

References

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