## PAPER

# Low Complexity Metric for Joint MLD in Overloaded MIMO System 

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#### Abstract

SUMMARY This paper presents a low complexity metric for joint maximum-likelihood detection (MLD) in overloaded multipleinput multiple-output (MIMO)-orthogonal frequency division multiplexing (OFDM) systems. In overloaded MIMO systems, a nonlinear detection scheme such as MLD combined with error correction coding achieves better performance than is possible with a single signal stream with higher order modulation. However, MLD incurs high computation complexity because of the multiplications in the selection of candidate signal points. Thus, a Manhattan metric has been used to reduce the complexity. Nevertheless, it is not accurate and causes performance degradation in overloaded MIMO systems. Thus, this paper proposes a new metric whose calculations involve only summations and bit shifts. New numerical results obtained through computer simulation show that the proposed metric improves bit error rate (BER) performance by more than 0.2 dB at the BER of $10^{-4}$ in comparison with a Manhattan metric. key words: overloaded MIMO, joint MLD


## 1. Introduction

Recently, mobile phones and wireless local area networks (LANs) are widely used all over the world. Wireless communication systems have to accommodate a large amount of data traffic. Multiple-input multiple-output (MIMO) has been proposed to realize wireless communication systems with larger capacity and better reliability [1], [2]. MIMO uses multiple antenna elements at both the transmitter and receiver.

However, a limited number of receive antennas can be implemented in a mobile terminal owing to its form factor. Thus, overloaded MIMO has been investigated [3]. In overloaded MIMO systems there are fewer receive antenna elements than transmit antenna elements. In the receivers of overloaded MIMO systems, nonlinear detection schemes such as maximum likelihood detection (MLD) are applied in conjunction with error correction coding and it achieves superior performance as compared to a single signal stream with higher order modulation [4], [5]. However, the computational complexity of MLD increases exponentially with the number of signal streams. In order to decrease the detection complexity in overloaded MIMO systems, a linear detection scheme has been proposed [6]-[9]. In these literatures, virtual channels are created by transforming the complex channel and signal matrices to the real value ma-

[^0]

Fig. 1 Overloaded MIMO-OFDM system.

## 2. System Description

### 2.1 Signal Model

The block diagram of an overloaded MIMO-OFDM system with joint MLD is shown in Fig. 1. Information bits are encoded by the turbo code and the coded bits are rearranged by the interleaver. The code rate is then adjusted through puncturing in the rate matching block. After puncturing $M$ coded bits are allocated to a $2^{M} \mathrm{QAM}$ symbol. The symbol on the $l$ th subcarrier, $S_{p}[l]$, is transmitted from the $p$ th transmit antenna. The OFDM signal transmitted from the $p$ th transmit antenna is given by

$$
\begin{equation*}
u_{p}[n]=\sum_{l=0}^{N-1} S_{p}[l] \exp \left(j \frac{2 \pi n l}{N}\right) \tag{1}
\end{equation*}
$$

where $n$ is the time index $(n=0,1, \ldots, N-1), N$ is the size of the inverse discrete Fourier transform (IDFT). A guard interval (GI) is then added by replicating the last part of the OFDM symbol. The transmit signal from the $p$ th antenna is written as

$$
\begin{equation*}
v_{p}(t)=\sum_{n=-N_{G I}}^{N-1} u_{p}[n] p_{t p}\left(t-n T_{s}\right) \tag{2}
\end{equation*}
$$

where $p_{t p}(t)$ is the impulse response of the transmit filter, $T_{s}$ is the sampling interval of the OFDM symbol, and $N_{G I}$ is the GI length. In the receiver side, the received signal at the $q$ th receive antenna is given by

$$
\begin{equation*}
y_{q}(t)=\sum_{p=1}^{N_{T}} y_{q p}(t)+n_{q}(t) \tag{3}
\end{equation*}
$$

where $n_{q}(t)$ is the noise at the $q$ th receive antenna and $y_{q p}(t)$ is the received signal from the $p$ th transmit antenna to the $q$ th receive antenna. $y_{q p}(t)$ is calculated by

$$
\begin{equation*}
y_{q p}(t)=\sum_{n=-N_{G I}}^{N-1} u_{p}[n] h_{q p}\left(t-n T_{s}\right) \tag{4}
\end{equation*}
$$

where $h_{q p}(t)$ is the impulse response of the channel between the $p$ th transmit and $q$ th receive antennas and it includes the transmit and receive filters. The received signal is converted
to digital samples by an A/D converter at the rate of $T_{s}$. Therefore, the received digital signal is given as

$$
\begin{equation*}
y_{q}[n]=y_{q}\left(n T_{s}\right) \tag{5}
\end{equation*}
$$

After removing the GI and taking the DFT of $N$ samples, the signal on the $l$ th subcarrier is expressed as

$$
\begin{align*}
Y_{q}[l] & =\sum_{n=0}^{N-1} y_{q}[n] \exp \left(j \frac{-2 \pi n l}{N}\right) \\
& =\sum_{p=1}^{N_{T}} H_{q p}[l] S_{p}[l]+N_{q}[l] \tag{6}
\end{align*}
$$

where $H_{q p}[l]$ is the frequency response between the $p$ th transmit antenna and the $q$ th receive antenna and $N_{q}[l]$ is the noise through the $q$ th receive antenna on the $l$ th subcarrier. It can be written in a matrix form as

$$
\begin{equation*}
\mathbf{Y}[l]=\mathbf{H}[l] \mathbf{S}[l]+\mathbf{N}[l] \tag{7}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathbf{Y}[l]=\left[\begin{array}{lll}
Y_{1}[l] & Y_{2}[l] & \ldots
\end{array} Y_{N_{R}}[l]\right]^{T},  \tag{8}\\
& \mathbf{H}[l]=\left[\begin{array}{ccc}
H_{11}[l] & \cdots & H_{1 N_{T}}[l] \\
\vdots & \ddots & \vdots \\
H_{N_{R} 1}[l] & \cdots & H_{N_{R} N_{T}}[l]
\end{array}\right],  \tag{9}\\
& \mathbf{S}[l]=\left[\begin{array}{lll}
S_{1}[l] & S_{2}[l] & \ldots
\end{array} S_{N_{T}}[l]\right]^{T},  \tag{10}\\
& \mathbf{N}[l]=\left[\begin{array}{llll}
N_{1}[l] & N_{2}[l] & \ldots & \left.N_{N_{R}}[l]\right]^{T}
\end{array}\right. \tag{11}
\end{align*}
$$

### 2.2 Calculation the LLR for Joint MLD

A log-likelihood ratio (LLR) is calculated for each coded bit in the systematic part of the codeward in joint MLD. The likelihood values for the $m$ th bit of the symbol in the $p$ th signal are given as follows;

$$
\begin{align*}
& D_{p m}^{1}=\sum_{\hat{\mathbf{S}}_{p m}^{1} \in\{\mathbf{S}\} b_{p m}=1} \exp \left(-\frac{1}{\sigma^{2}}\left\|\mathbf{Y}[l]-\mathbf{H}[l] \hat{\mathbf{S}}_{p m}^{1}\right\|^{2}\right),  \tag{12}\\
& D_{p m}^{0}=\sum_{\hat{\mathbf{S}}_{p m}^{0} \in\{\mathbf{S}\} b_{p m}=0} \exp \left(-\frac{1}{\sigma^{2}}\left\|\mathbf{Y}[l]-\mathbf{H}[l] \hat{\mathbf{S}}_{p m}^{0}\right\|^{2}\right), \tag{13}
\end{align*}
$$

where $\sigma^{2}$ is the noise variance, $\hat{\mathbf{S}}_{p m}^{1}$ or $\hat{\mathbf{S}}_{p m}^{0}$ is the vector
of candidate $N_{T}$ symbols in which the $m$ th coded bit of a $2^{M}$ QAM symbol from the $p$ th transmit antenna is " 1 " or ${ }^{\prime} 0^{\prime \prime}, \mathbf{S}=\left[\begin{array}{llll}S_{1} & S_{2} & \ldots & S_{N_{T}}\end{array}\right]^{T}$ is the candidate symbol vector, $\{\mathbf{S}\}_{b_{p m}=1}$ or $\{\mathbf{S}\}_{b_{p m}=0}$ is the set of the symbol vectors in which the $m$ th coded bit of the $p$ th $2^{M}$ QAM symbol, $S_{p}$, is " 1 " or " 0 ", and $D_{p m}^{1}$ or $D_{p m}^{0}$ is the sum of the likelihood values for the $m$ th coded bit of " 1 " or " 0 " in the $p$ th symbol, respectively. The LLR is calculated as,

$$
\begin{equation*}
L\left(b_{p m} \mid \mathbf{Y}[l]\right)=\log \frac{D_{p m}^{1}}{D_{p m}^{0}} \tag{14}
\end{equation*}
$$

where $L\left(b_{p m} \mid \mathbf{Y}[l]\right)$ is the LLR of the $m$ th bit of the symbols on the $l$ th subcarrier transmitted from the $p$ th antenna.

### 2.3 Approximation of LLR

The approximation of the LLR with the dominant terms has been given in [14]. The approximated LLR is given as

$$
\begin{align*}
L\left(b_{p m} \mid \mathbf{Y}[l]\right) & \approx\left(-\frac{1}{\sigma^{2}}\left\|\mathbf{Y}[l]-\mathbf{H}[l] \overline{\mathbf{S}}_{p m}^{1}\right\|^{2}\right) \\
& -\left(-\frac{1}{\sigma^{2}}\left\|\mathbf{Y}[l]-\mathbf{H}[l] \overline{\mathbf{S}}_{p m}^{0}\right\|^{2}\right) \tag{15}
\end{align*}
$$

where $\overline{\mathbf{S}}_{p m}^{1}$ or $\overline{\mathbf{S}}_{p m}^{0}$ is one of the candidate symbol vectors in $\left\{\hat{\mathbf{S}}_{p m}^{1}\right\}$ or $\left\{\hat{\mathbf{S}}_{p m}^{0}\right\}$ and it gives the minimum Euclidean metric from $\mathbf{Y}[l]$ as follows;

$$
\begin{align*}
& \overline{\mathbf{S}}_{p m}^{1}=\underset{\hat{\mathbf{S}}_{p m}^{1} \in\left\{\mathbf{S}_{b_{p p m}=1}\right.}{\arg \min } \sum_{q=1}^{N_{R}}\left\{\left(\mathfrak{R}\left[E_{q p m}^{1}[l]\right]\right)^{2}+\left(\mathfrak{J}\left[E_{q p m}^{1}[l]\right]\right)^{2}\right\},  \tag{16}\\
& \overline{\mathbf{S}}_{p m}^{0}=\underset{\hat{\mathbf{S}}_{p m}^{0} \in\left\{\mathbf{S}_{\}_{p p m}=0}\right.}{\arg \min } \sum_{q=1}^{N_{R}}\left\{\left(\mathfrak{R}\left[E_{q p m}^{0}[l]\right]\right)^{2}+\left(\mathfrak{J}\left[E_{q p m}^{0}[l]\right]\right)^{2}\right\}, \tag{17}
\end{align*}
$$

where $E_{q p m}^{1}[l]$ or $E_{q p m}^{0}[l]$ is the difference between the coordinates of the received signal and the candidate signal point in which the $m$ th coded bit of the $p$ th signal stream is " 1 " or " 0 " and these are calculated by the followings;

$$
\begin{align*}
& E_{q p m}^{1}[l]=Y_{q}^{1}[l]-\mathbf{H}_{q p}^{1}[l] \hat{\mathbf{S}}_{p m}^{1}[l],  \tag{18}\\
& E_{q p m}^{0}[l]=Y_{q}^{0}[l]-\mathbf{H}_{q p}^{0}[l] \hat{\mathbf{S}}_{p m}^{0}[l], \tag{19}
\end{align*}
$$

where $\mathbf{H}_{q p}[l]=\left[H_{q 1}[l] H_{q 2}[l] \ldots H_{q N_{T}}[l]\right]^{T}$.
This approximation omits the calculation of exponential terms. However, it is required to determine the dominant terms by calculating the Euclidean metric.

### 2.4 Metric Calculation without Multiplication

### 2.4.1 Conventional Metric

In the joint MLD, the metrics between the received signal
and all the candidate signal points are required to be calculated. It is necessary to calculate the Euclidean metrics for all the combinations of the encoded symbols from the $N_{T}$ signal streams. The number of multiplications increase exponentially with the number of signal streams. In order to reduce the complexity, a Manhattan metric has been introduced in MLD [13].

The Manhattan metric between the received signal and the candidate signal points are calculated and the candidate symbol vector with the minimum among them is selected as follows;

$$
\begin{align*}
& \tilde{\mathbf{S}}_{p m}^{c 1}=\underset{\hat{\mathbf{S}}_{p m}^{1} \in\{\mathbf{S}\}_{b_{p m}=1}}{\arg \min } \sum_{q=1}^{N_{R}}\left(\left|\mathfrak{R}\left[E_{q p m}^{1}[l]\right]\right|+\left|\mathfrak{I}\left[E_{q p m}^{1}[l]\right]\right|\right),  \tag{20}\\
& \tilde{\mathbf{S}}_{p m}^{c 0}=\underset{\hat{\mathbf{S}}_{p m}^{0} \in\{\mathbf{S}\}_{b_{p m}=0}}{\arg \min } \sum_{q=1}^{N_{R}}\left(\left|\mathfrak{R}\left[E_{q p m}^{0}[l]\right]\right|+\left|\mathfrak{I}\left[E_{q p m}^{0}[l]\right]\right|\right), \tag{21}
\end{align*}
$$

where $\tilde{\mathbf{S}}_{p m}^{c 1}$ or $\tilde{\mathbf{S}}_{p m}^{c 0}$ is the candidate symbol vector from $\left\{\hat{\mathbf{S}}_{p m}^{1}\right\}$ or $\left\{\hat{\mathbf{S}}_{p m}^{0}\right\}$ with the smallest Manhattan metric, and $E_{q p m}^{1}[l]$ and $E_{q p m}^{0}[l]$ are calculated by Eqs. (18) and (19). The approximated LLR is calculated with the following equations.

$$
\begin{align*}
& \tilde{D}_{p m}^{1}=\frac{1}{\sigma^{2}}\left\|\mathbf{Y}[l]-\mathbf{H}[l] \tilde{\mathbf{S}}_{p m}^{1}\right\|^{2}  \tag{22}\\
& \tilde{D}_{p m}^{0}=\frac{1}{\sigma^{2}}\left\|\mathbf{Y}[l]-\mathbf{H}[l] \tilde{\mathbf{S}}_{p m}^{0}\right\|^{2}  \tag{23}\\
& L\left(b_{p m} \mid \mathbf{Y}[l]\right) \approx \tilde{D}_{p m}^{1}-\tilde{D}_{p m}^{0} \tag{24}
\end{align*}
$$

### 2.4.2 Proposed Metric

The proposed metric consists of the two terms, $\| \mathscr{R}\left[E_{q p m}^{*}\right] \mid+$ $\mid \mathfrak{I}\left[E_{q p m}^{*}\right] \|$ and $\left\|\Re\left[E_{q p m}^{*}\right]-\mathfrak{J}\left[E_{q p m}^{*}\right]\right\|(*=" 1 "$ or " 0 "), for the selection of the candidate signal vector among $\left\{\hat{\mathbf{S}}_{p m}^{1}\right\}$ or $\left\{\hat{\mathbf{S}}_{p m}^{0}\right\}$. These terms are combined with corresponding coefficients. The curves of the metrics are drawn when the difference of the coordinates of the signal points is given as $e^{i \theta}$ when $\theta$ is from 0 to $\pi / 2$ in Fig. 2. Here, the Euclidean metric is constant and its value is normalized to $\left|e^{i \theta}\right|=1$. For the same difference of the coordinates, the Manhattan metric shows the maximum value of 1.41 at $\theta=\pi / 4$. The proposed metric achieves a smaller error from the Euclidean metric than that of the Manhattan metric.

The candidate symbol vector with the minimum metric is selected as the following equations.

$$
\begin{align*}
\tilde{\mathbf{S}}_{p m}^{n 1}=\underset{\hat{\mathbf{S}}_{p m}^{1} \in\left\{\mathbf{S}_{b_{p m}=1}^{\arg \min }\right.}{ } & \sum_{q=1}^{N_{R}}\left\{C_{+}\left(\left|\mathfrak{R}\left[E_{q p m}^{1}[l]\right]\right|+\left|\mathfrak{I}\left[E_{q p m}^{1}[l]\right]\right|\right)\right. \\
& \left.+C_{-}\left(\| \mathfrak{R}\left[E_{q p m}^{1}[l]\right]\left|-\left|\mathfrak{I}\left[E_{q p m}^{1}[l]\right]\right|\right|\right)\right\}, \tag{25}
\end{align*}
$$



Fig. 2 Conventional and proposed metrics.

$$
\begin{align*}
\tilde{\mathbf{S}}_{p m}^{n 0}=\underset{\hat{\mathbf{S}}_{p m}^{0}\left[\mid \mathbf{S}_{l_{p m}}=0\right.}{\arg \min } & \sum_{q=1}^{N_{R}}\left\{C_{+}\left(\left|\mathfrak{R}\left[E_{q p m}^{0}[l]\right]\right|+\left|\mathfrak{J}\left[E_{q p m}^{0}[l]\right]\right|\right)\right. \\
& +C_{-}\left(\| \mathbb{R}\left[E_{q p m}^{0}[l]\right]\left|-\left|\mathfrak{S}\left[E_{q p m}^{0}[l]\right]\right|\right)\right\}, \tag{26}
\end{align*}
$$

where $\tilde{\mathbf{S}}_{p m}^{n 1}$ or $\tilde{\mathbf{S}}_{p m}^{n 0}$ is the candidate symbol vector from $\left\{\hat{\mathbf{S}}_{p m}^{1}\right\}$ or $\left\{\hat{\mathbf{S}}_{p m}^{0}\right\}$ with the smallest metric, $E_{q p m}^{1}[l]$ and $E_{q p m}^{0}[l]$ are calculated by Eqs. (18) and (19), and $C_{+}$and $C_{-}$are the coefficients that decide the ratio of the two terms, $\| \mathfrak{R}\left[E_{q p m}^{*}\right] \mid+$ $\mid \mathfrak{I}\left[E_{q p m}^{*}\right] \|$ and $\left\|\mathfrak{R}\left[E_{q p m}^{*}\right]-\mathfrak{I}\left[E_{q p m}^{*}\right]\right\|$. For the reduction of the computationally complexity, $C_{+}$and $C_{-}$are given with the following condition.

$$
\begin{equation*}
C_{+}=1-C_{-}=\sum_{k} \frac{C_{k}}{2^{k}} \tag{27}
\end{equation*}
$$

where $k$ is an integer and $C_{k}=\{-1,0,1\}$. It is then possible to substitute the multiplication of the coefficient through bit shifts and summations. With the selected candidate symbol vectors the likelihood ratio is then calculated by Eq. (15).

## 3. Numerical Results

### 3.1 Simulation Conditions

The simulation conditions are shown in Table 1. A turbo code with 8 state memory is applied as the error-correction code [15]. The base rate of the code is $1 / 3$ and the code rate is selected from $1 / 3,1 / 2,5 / 6$ through puncturing. The interleaver size is fixed to 4800 . The number of the transmit antennas is selected from 2 to 4 while the number of the receive antennas is fixed to 2 . Each signal stream is modulated with QPSK, 16QAM, 64QAM and multiplexed through OFDM. The channel bandwidth is 2.5 MHz and subcarrier spacing is 15 kHz . The number of subcarriers is 256 while the number of data subcarriers is 151 . The sampling frequency is then 3.84 MHz . The guard interval of the 1 st symbol is $5.21 \mu \mathrm{~s}$, and the $2 \mathrm{nd}-7$ th symbols are $4.69 \mu \mathrm{~s}$. 6 Taps GSM-TU is assumed as a channel model [16]. Channel estimation in the receiver is assumed to be ideal. The

Table 1 Simulation conditions.

| Error Correction Coding | Turbo Code |
| :--- | :--- |
| Code Rate | $1 / 3,1 / 2,5 / 6$ |
| Interleave Size | 4800 |
| Modulation Scheme | QPSK,16QAM,64QAM <br> /OFDM |
| Number of Signal Streams | $2 \sim 4$ |
| Number of | $2 \sim 4$ |
| Transmit Antennas | 2 |
| Number of |  |
| Receive Antennas | 2.5 MHz |
| Channel Bandwidth | 15 kHz |
| Subcarrier Spacing | 256 |
| Number of Subcarriers | 151 |
| Number of Data Subcarriers | 3.84 MHz |
| Sampling Frequency | $5.21 \mu \mathrm{~s}(1$ st symbol $)$ |
| Guard Interval | $4.69 \mu \mathrm{~s}(2 \mathrm{nd}-7 \mathrm{th}$ symbols $)$ |
| Decoding Algorithm | Log-MAP |
| Number of | 8 |
| Decoding Iterations | 6 Taps GSM-TU |
| Channel Model | Ideal |
| Channel Estimation | $\left(\frac{21}{3}, \frac{11}{32}\right)$ |
| Scheme I $:\left(C_{+}, C-\right)$ | $\left(\frac{11}{16}, \frac{5}{16}\right)$ |
| Scheme II $:\left(C_{+}, C-\right)$ | $\left(\frac{5}{8}, \frac{3}{8}\right)$ |
| Scheme III $:\left(C_{+}, C-\right)$ | $(2,1)$ |
| Scheme IV $:\left(C_{+}, C-\right)$ | $9.6 \times 10^{6}$ |
| Number of Bits |  |

decoding algorithm is log-MAP and the number of decoding iterations is 8 . For each plot, $9.6 \times 10^{6}$ bits are transmitted.

### 3.2 Coefficients of Proposed Metric

The coefficients for the proposed metric are selected from $\left(C_{+}, C-\right)=\left(\frac{21}{32}, \frac{11}{32}\right),\left(\frac{11}{16}, \frac{5}{16}\right),\left(\frac{5}{8}, \frac{3}{8}\right),(2,1)$. They are referred to as the proposed scheme I, II, III, and IV. These sets of the coefficients for the proposed metric I, II, and III correspond to the maximum value of $k$ in Eq. (27). The coefficients for the proposed metric I, II, and III are calculated with $k$ of up to 5,4 , and 3 . The coefficients, $\left\{C_{k}\right\}$, are selected so that the resultant metric is close to the Euclidean metric with respect to the mean error. The metrics for the maximum $k$ of 1 and 2 are excluded since they are not close to the Euclidean metric than the proposed metrics in terms of the parameters shown in Table 2. As for the metrics with the maximum $k$ of more than 6 are also excluded since they require more bit shifts and summations that lead to larger complexity. The coefficients of $\left(C_{+}, C_{-}\right)=(2,1)$ are selected for its simplicity as it requires only one bit shift though they do not satisfy the condition of $C_{+}=1-C_{-}$. The curves of the proposed metrics are presented in Fig. 3. For the case of the proposed scheme IV, the coefficients of $\frac{1}{3}(2,1)$ are used to draw the curve though $1 / 3$ is omitted at the selection of the candidate signal point for complexity reduction. The mean, the mean square, and the maximum difference from the Euclidean metric is shown in Table 2. It is clear that the mean and the maximum of the errors with the proposed metric are smaller than those of the Manhattan metric.

Table 2 Accuracy of metrics.

| Method | Mean | Mean Square | Max. |
| :--- | :--- | :--- | :--- |
| Manhattan Metric | 0.2716 | 0.0896 | 0.4142 |
| Proposed Metric I | 0.0319 | 0.0013 | 0.0697 |
| Proposed Metric II | 0.0422 | 0.0023 | 0.0680 |
| Proposed Metric III | 0.0318 | 0.0017 | 0.1137 |
| Proposed Metric IV | 0.0342 | 0.0014 | 0.0550 |



Fig. 3 Curves of proposed metric I, II, III, IV.

### 3.3 BER Performance

The bit error rate (BER) versus $E_{b} / N_{0}$ with 3 signal streams for the code rates of $1 / 3,1 / 2$, and $5 / 6$ on the 6 Taps GSMTU channel is shown in Figs. 4-6, respectively. The performance with the LLR approximation deteriorates by 0.4 dB in Fig. 4, by 0.2 dB in Fig. 5, and by 0.05 dB in Fig. 6 at the BER of $10^{-4}$ as compared to that without approximation. The BER with the Manhattan metric further increases the required $E_{b} / N_{0}$ by $0.2-0.4 \mathrm{~dB}$. On the other hand, the proposed metric improves the BER performance by more than 0.2 dB in Fig. 4, 0.2 dB in Fig. 5, and 0.05 dB in Fig. 6 at the BER of $10^{-4}$ in comparison with the Manhattan metric.

The required $E_{b} / N_{0}$ at $\mathrm{BER}=10^{-4}$ versus the code rate with 3 signal streams on the 6 Taps GSM-TU channel is shown in Fig. 7. It is shown that the amount of the degradation on the required $E_{b} / N_{0}$ reduces as the code rate grows in Fig. 7. This is because the error-correcting capability of the turbo decode deteriorates as the code rate increases. The proposed metrics I, II, IV are relatively better than the proposed metric III because the maximum difference is smaller.

The required $E_{b} / N_{0}$ at $\mathrm{BER}=10^{-4}$ versus the number of signal streams with the code rate of $1 / 3$ on the 6 Taps GSMTU channel is shown in Fig. 8. It is shown that the amount of


Fig. 4 BER vs. $E_{b} / N_{0}$ (3 signal streams, code rate of 1/3, QPSK, 6 Taps GSM-TU).


Fig. 5 BER vs. $E_{b} / N_{0}(3$ signal streams, code rate of 1/2, QPSK, 6 Taps GSM-TU).
the performance degradation with the LLR approximations enlarges as compared to that without approximation when the number of signal streams increases. The reason is that the error of the metric causes the miss selection of the candidate signal point more often as the number of signal streams increases. The proposed metrics achieve closer performance to that with the Euclidean metric than the Manhattan metric when the number of signal streams is up to 4 .

The BER curves versus $E_{b} / N_{0}$ with 3 signal streams with 16QAM or 64QAM are shown in Figs. 9 and 10, respectively. The code rate is set to $1 / 2$. It is clear that the


Fig. 6 BER vs. $E_{b} / N_{0}$ (3 signal streams, code rate of 5/6, QPSK, 6 Taps GSM-TU).


Fig. 7 Required $E_{b} / N_{0}$ for BER $=10^{-4}$ vs. code rate (3 signal streams, QPSK, 6 Taps GSM-TU).
performance with 16QAM and 64QAM shows the similar tendency as that with QPSK modulation in Fig. 5. The performance improvements with the proposed metric are about 0.1 dB as compared to those with the Manhattan metric in both Figs. 9 and 10.

### 3.4 Complexity

The numbers of multiplications, summations, and bit shifts for each metric are shown in Table 3. The numbers of multiplications in the proposed metrics and the Manhattan met-


Fig. 8 Required $E_{b} / N_{0}$ at $\mathrm{BER}=10^{-4}$ vs. number of signal streams (code rate of $1 / 3$, QPSK, 6 Taps GSM-TU).


Fig. 9 BER vs. $E_{b} / N_{0}$ (3 signal streams, code rate of 1/2, 16QAM, 6 Taps GSM-TU).
ric per receive antenna are in proportion to the number of transmit signal streams while it increases exponentially in the Euclidean metric. On the other hand, the numbers of summations per receive antenna increase exponentially to the number of the signal streams in all the metrics. The proposed metric requires from $5 / 3$ to 3 times of summations as compared to the Manhattan metric. The proposed metric also needs additional bit shift operations though their complexity is independent of the number of signal streams.


Fig. 10 BER vs. $E_{b} / N_{0}$ ( 3 signal streams, code rate of $1 / 2$, 64QAM, 6 Taps GSM-TU).

Table 3 Complexity of metrics.

| Method | Multiplications | Summations | Bit <br> Shifts |
| :--- | :--- | :--- | :--- |
| Euclidean <br> Metric | $2^{2 N_{T} N_{R}}$ | $2^{2 N_{T} N_{R}}$ | 0 |
| Manhattan <br> Metric | $2 \times 2 N_{T} N_{R}$ | $3 \times 2^{2 N_{T}} N_{R}$ | 0 |
| Proposed <br> Metric I | $2 \times 2 N_{T} N_{R}$ | $9 \times 2^{2 N_{T}} N_{R}$ | $6 N_{R}$ |
| Proposed <br> Metric II | $2 \times 2 N_{T} N_{R}$ | $8 \times 2^{2 N_{T}} N_{R}$ | $4 N_{R}$ |
| Proposed <br> Metric III | $2 \times 2 N_{T} N_{R}$ | $7 \times 2^{2 N_{T}} N_{R}$ | $4 N_{R}$ |
| Proposed <br> Metric IV | $2 \times 2 N_{T} N_{R}$ | $5 \times 2^{2 N_{T} N_{R}}$ | $N_{R}$ |

## 4. Conclusions

This paper proposes a low complexity metric that is calculated only through summations and bit shifts. The proposed metric uses the subtraction of the differences on the real and imaginary parts between a received signal point and a candidate signal constellation point as well as the sum of those differences.

The numerical results obtained through computer simulation show that the proposed metric improves the BER performance by more than 0.2 dB at the BER of $10^{-4}$ in comparison with that with the Manhattan metric. The performance degradation with the proposed metric is enhanced at low code rates while it is almost the same performance as that without approximation with the higher code rate. The proposed metric achieves better performance than that with the Manhattan metric even though the number of signal streams increases.

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